Uncertainty in Marine Structural Strength Due to Variability in Geometry and Material Properties

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**Naval Surface Warfare Center**
**Carderock Division**
West Bethesda, MD 20817-5700

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Survivability, Structures, and Materials Directorate

Technical Report

# Uncertainty in Marine Structural Strength Due to Variability in Geometry and Material Properties

by

Bilal M. Ayyub, Rafi Muhanna and Daniel D. Bruchman



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1. Reference (a) directed the Naval Surface Warfare Center, Carderock Division (NSWCCD) to develop prototype computational methodology for reliability assessment of continuum structures. Enclosure (1) describes the parametric finite element analysis, reliability assessment and ancillary computer programs developed in the course of this task. Geometric and material uncertainties were considered in the finite element model, and examples are included to demonstrate and test the methodology. A portion of this work was performed under contract with the University of Maryland.

2. Comments or questions may be referred to the principal investigator, Mr. Daniel D. Bruchman, Code 651; telephone (301) 227-4113; e-mail, bruchman@oasys.dt.navy.mil.

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This report describes the development of a prototype computational methodology for the reliability assessment of continuum structures using finite element analysis with instability failure modes. Geometric and material uncertainties were considered in the finite element model. The reliability assessment was performed using Monte Carlo simulations with variance reduction techniques that consisted of the conditional expectation method. According to Monte Carlo methods, the applied load was randomly generated, and finite element analysis was used to predict the response of the structure under the generated loads in the form of a deformation field. A simulation procedure was applied to compare the response with a specified failure definition and failures were then counted. By repeating the simulation procedure several times, the failure probability defined by the specified failure definition was estimated as the failure fraction of simulation repetitions. A parametric study was performed for a stiffened panel strength to illustrate the procedure.

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## SUMMARY

This report describes a prototype computational methodology for reliability assessment of continuum structures using finite element analysis with instability failure modes. Examples are included to illustrate and test the methodology. Geometric and material uncertainties were considered in the finite element model. A computer program was developed that integrates uncertainty formulations to create a finite element input file, and conducts the reliability assessment using FORTRAN. A commercial finite element package was used as a basis for strength assessment. Another computer program was used to control and manage the data flow between the reliability assessment routine and the finite element software. A parametric study for a stiffened panel strength was also carried out.

The finite element model was based on the 8-node doubly curved shell element, which can provide the non-linear behavior prediction of the stiffened panel. The mesh was designed to ensure the convergence of eigenvalue estimates. Failure modes were predicted on the basis of elastic non-linear analysis using the finite element model.

Reliability assessment was performed using Monte Carlo simulation with variance reduction techniques that consisted of the conditional expectation method. According to Monte Carlo methods, the applied load was randomly generated, and finite element analysis was used to predict the response of the structure under the generated loads in the form of a deformation field. A crude simulation procedure was applied to compare the response with a specified failure definition, and failures were then counted. By repeating the simulation procedure several times, the failure probability according to the specified failure definition was estimated as the failure fraction of simulation repetitions. Alternatively, conditional expectation was used to estimate the failure probability in each simulation cycle in this study, then the average failure probability and its statistical error were computed.

The developed method is expected to have significant impact on the reliability assessment of structural components and systems; more specifically, the safety and reliability evaluation of continuum structures, the formulation of associated design criteria, evaluation of important variables that influence failures, the possibility of revising some codes of practice, reduction of the number of required costly experiments in structural testing, and the safety evaluation of existing structures for the purpose of life extension. The impact of this study can extend beyond structural reliability into the generalized field of engineering mechanics.


## ADMINISTRATIVE INFORMATION


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# 1. INTRODUCTION AND OBJECTIVES

The main objective of engineering design is to ensure the safety and performance of an engineering system for a given period of time and for specified loading conditions. The absolute safety of a system cannot be guaranteed due to the uncertainties involved in structures, their loading and behavior. However, through reliability methods, the system probability of failure of the system can be limited to an acceptable level. In order to control the reliability of structural systems, the failure probability needs first to be estimated and assessed. In the area of structural reliability assessment, structural systems with discrete components, such as trusses and frames, have received more attention and research than continuum structures. However, dealing with a structure as a continuum can provide a generalized basis of analysis. Such a modeling approach can lead to a better understanding of its behavior and analysis of its performance under different conditions. Also, it can deal with uncertainties in geometric and material properties for conventional and innovative materials, as well as uncertainties in failure definition and failure modes. Therefore, a need exists to adapt and use state-of-the art reliability assessment methods to deal with continuum structures.

Reliability assessment requires the knowledge of structural strength which is associated with different types of geometric and material uncertainties. In general, uncertainties in structural behavior and performance may be associated with physical phenomena that are inherently random or with predictions of reality performed under conditions of incomplete or inadequate information. Therefore, uncertainty may be associated with inherent variability of a physical process or with imperfection in the modeling of a physical process. Moreover, prediction or modeling error may contain two components, the systematic component and the random component. In measurement theory, these are known as the *systematic error* and *random error*, respectively. From a practical standpoint, inherent variability is essentially a state of nature and the resulting uncertainty may not be controlled or reduced. The uncertainty associated with prediction or modeling error may be reduced through the use of more accurate models or acquisition of additional data. Uncertainties associated with inherent variability, as well as random error, can be expressed in terms of coefficients of variation. These uncertainty measures are needed to accurately assess probabilities of interest. Methods for evaluating uncertainty measures including biasedness depend on the form of the available data and information (Ang and Tang, 1984). Failure or survival of real structures according to the different serviceability or strength criteria are continuous and gradual rather than crisp and abrupt. Consequently, failure or survival definitions are accompanied with uncertainty as discussed in Section 2.1. The

uncertainty in a failure definition can be considered to be of the vagueness type. Fuzzy set theory can be used to deal with this type as demonstrated by Ayyub and Lai (1992).

Simulation methods can be used for structural reliability assessment. In their fundamental form, applied loads to a structure can be randomly generated, then finite element analysis can be used to predict the response of the structure under a combined state of the generated loads. The response can be in the form of a stress field, strain field, or deformation field, or their combinations. Using a crude simulation procedure, the response needs to be compared with a specified failure definition, and failures counted. By repeating the simulation procedure several times, the failure probability according to the specified failure definition can be estimated as the failure fraction of simulation repetitions.

The objectives of this study are to develop a prototype computational procedure for reliability assessment of a continuum structure using finite element analysis with instability failure modes, and to test the procedure. A state-of-the art Finite Element (FE) commercial package is used in this study to assess the strength of a continuum structure. In addition, parametric analyses for the strength and reliability of a stiffened panel due to changes in the uncertainties in geometric and material properties are performed.

# 2. METHODOLOGY

## 2.1. General Description

The development of a methodology for the reliability assessment of continuum ship structural components or systems requires the consideration of the following three components: (1) loads, (2) structural strength, and (3) methods of reliability analysis. Also, the reliability analysis requires the probabilistic characteristics of the operational-sea profile of a ship, failure modes, and failure definitions. A reliability assessment methodology can be developed in the form of the following modules: operational-sea profile and loads; nonlinear structural analysis; extreme analysis and stochastic load combination; failure modes, their load effects, load combinations, and structural strength; library of probability distributions; reliability assessment methods; uncertainty modeling and analysis; failure definitions; and system analysis. Each module can be independently investigated and developed, although some knowledge about the details of other modules is needed for the development of a module. These modules are described by Ayyub, Beach and Packard (1995).

Prediction of structural failure modes of continuum ship structural components or systems requires the use of nonlinear structural analysis. Also, the identification, i.e., recognition, and proper classification of failures based on a structural response within the simulation process needs to be performed based on deformations. Therefore, failure definitions need to be expressed using deformations rather than forces or stresses. The process of failure classification and recognition needs to be automated in order to facilitate its use in a simulation algorithm for structural reliability assessment. Figure 1 shows a procedure for an automated failure classification that can be implemented in a simulation algorithm for reliability assessment. The failure classification is based on matching a deformation or stress field with a record within a knowledge base of response and failure classes. In cases of no match, a list of approximate matches is provided, with assessed applicability factors. The user can then be prompted for any

changes to the approximate matches and their applicability factors. In the case of a poor match, the user can have the option of activating the failure recognition algorithm shown in Fig. 2 to establish a new record in the knowledge base. The adaptive or neural nature of this algorithm allows the updating of the knowledge base of responses and failure classes. The failure recognition and classification algorithm shown in Fig. 2 evaluates the impact of the computed deformation or stress field on several systems of a ship. The impact assessment includes evaluating the remaining strength, stability, repair criticality, propulsion and power systems, combat systems, and hydrodynamic performance. The input of experts in ship performance is needed to make these evaluations using either numeric or linguistic measures. Then, the assessed impacts need to be aggregated and combined to obtain an overall failure recognition and classification within the established failure classes. The result of this process is then used to update the knowledge base.

## 2.2. Finite Element Method

The development of a general Finite Element (FE) computational approach for continuum mechanics has dominated the efforts of researchers over the past three decades. The efforts have followed three main courses: (1) the so-called degenerated solid approach, which finds its point of departure in the work of Ahmad, Irons and Zienkiewicz (1970); (2) the natural approach of Argyris et al. (1968 and 1969), which illustrated their development of the SHEBA family of shell elements; and (3) recently the shell theory which was initially presented in the paper of Simo and Fox (1989).

Stochastic finite element methods were also developed to calculate the stochastic response and reliability of linear and non-linear engineering systems by first-order or second-order reliability methods (FORM and SORM). In these methods geometric and materials property randomness are incorporated into a finite element formulation using approximate moment-based formulations. For example, the works of Contreras (1980), Hisada and Nakagiri (1985), Liu, et al (1986), Liu, et al (1988), Mochio, et al (1985), Sarras, et al (1993), Spanos, et al (1989), Vanmarcke and Grigoriu (1984), and Vanmarcke, et al (1986) provide such formulations and examples. Faravelli (1988) describes the use of finite element methods for reliability assessment based on response surface techniques.

Finite element methods have been used extensively in many areas such as vibration and dynamic response, buckling and postbuckling analysis with or without geometrical and material nonlinearities, thermal effect, fluid-structural interaction, aeroelasticity, structure-acoustics interaction, fracture, laminated composites, wave propagation, structural dynamics and control interaction of aircraft and space structures, random dynamic response, and others. Therefore, any development in the field of reliability assessment of continuum structures can have some impact on these areas.

### 2.2.1. Selecting a FE-package

The purpose of a finite element analysis is to predict the response of a model to some form of external loading, or to some nonequilibrium conditions. Reliability assessment of a structure requires the prediction of the structural response due to extreme loading. Generally, this response reflects both linear and/or non-linear behavior in geometry and material of the structure. The structural behavior could be expressed by local or global deformations, stress fields, strain

fields, static or dynamic criteria. Therefore, any selected FE-package for estimating the structural response needs to be capable of calculating all relevant values with acceptable accuracy levels. In addition, the output files of the selected FE-package should be in a suitable format to facilitate finding and reading strength prediction values for failure recognition and ultimately reliability assessment.

### 2.3. Random Number Generation

Numerical simulation is associated with random number generation of random variables in accordance with their respective or prescribed probability distributions. The random generation of each random variable can be accomplished systematically by first generating uniformly distributed random numbers between 0 and 1, and through appropriate transformations obtaining the corresponding random values with the specified probability distribution. The random generation efforts in a simulation process include generating random numbers (e.g., by a pseudo random number generator) and generating random variates through transformation (e.g., by substituting the generated random numbers into the inverse cumulative distribution functions of the respective random variables). A random number generator (Press et al. 1992) is then used to produce random numbers for all random variables.

Ayyub and Chao (1994) developed a library of subroutines for generating random variables with any of the following distribution types: uniform, triangular, normal, lognormal, Poisson, binomial, geometric, exponential, Rayleigh, extreme value type I - largest, extreme value type II - largest, extreme value type III - smallest (Weibull), and Gamma, experimental distributions. For each distribution, the following computations are needed: probability density values, or probability mass value; cumulative distribution value; inverse of cumulative distribution; random generation; and relationship between its parameters and moments. This library of subroutines was used in this study.

### 2.4. Random Variables in FEM

Finite element methods have a flexibility to represent different types of geometric and material uncertainties. In this study and as an illustrative example, a stiffened panel as shown in Fig. 3 was analyzed for different types of variability. The panel consisted of a thin rectangular plate with parallel stiffeners spaced at equal distances. The finite element model was based on the 8-node doubly curved shell element which can give the non-linear behavior prediction of the stiffened panel. The mesh, as shown in Fig. 4, was designed to ensure the convergence of eigenvalue estimates of its buckling loads. A systematic variability analysis of the panel required the hypothetical division of the panel into three levels: the plate level (level-0), the web level (level-1) and the flange level (level-2). The dimensions of the plate, the web, and the flange sides were denoted as L0i, L1i and L2i (i = 1,...., 4), respectively. Geometric and material uncertainties in the form of random variables were accounted for in the three levels, i.e., plate, web, and flange levels. The underlying geometric and strength random variables were defined based on reported values, practical aspects, and judgment (Ayyub, et al 1994, Ellingwood, et al 1980).

The plate level as shown in Fig. 5, included three types of variability: its sides' dimensions, out-of plane distortion, and thickness. The variability in the sides' dimensions for the plate (Fig. 5) was modeled using random variables for the first and third sides of the plate, i.e., sides L01

and L03. Such a variability subjects the stiffeners to possible out of plane moments. The second variability, as shown in Fig. 5, is the plate out-of-plane distortion modeled using three random variables for the z coordinates of its three corners, namely, zP02, zP03 and zP04. The third variability is the plate thickness and it is represented by a single, random variable t0.

The variability at the web level is shown in Fig. 6 to consist of four types: web tilting, web height, web bowing, and web thickness. The web tilting of Fig. 6 was represented by two random variables X21i and X23i per web, where (i = 1,....., number of stiffeners). These random variables model the horizontal deviation of the web top from the vertical position. The variability in the web height, as shown in Fig 6, was represented by two random variables (L11i and L13i) per web, where (i = 1,....., number of stiffeners). The web bowing is shown in Fig. 7 and was represented by the random variable XB0i, where (i = 1,....., number of stiffeners). This variable represents the possible out-of-plane deviation of the mid-web from the original plane position. The fourth variability uses the web thickness that is represented by the random variable t1.

The variability at the flange level is shown in Fig. 8 with its three types: flange tilting, flange height, and flange thickness. The flange tilting, as shown in Fig. 8, was represented by two random variables (Z2i0 and Z2iL) per flange, where (i = 1,....., number of stiffeners). These random variables represent the vertical deviation of the flange front and back sides from the horizontal position assuming that the web meets the flange at mid-width. The variability in the flange width as shown in Fig 8 was represented by two random variables (L21i and L23i) per flange, where (i = 1,....., number of stiffeners). The third variability uses the flange thickness that is represented by the random variable t2.

### 2.5. Reproduction of FE Input Files

Reliability assessment based on finite element analysis requires automated access to the input and output of a finite element package. Most general-purpose, finite element packages restrict automated changes in their input files in a sequence of runs as needed especially in simulation-based reliability assessment methods. Usually input files are prepared for specific models with deterministic values for the model variables. FE package uses in a simulation process requires the reproduction of the FE input file corresponding to the generated random variable in a simulation cycle. Figure 9 shows a developed procedure for FE input file reproduction in the *i*th simulation cycle. The block corresponding to FE input file reproduction consists of a main program which calls a random number generator (Press, et al 1992), a distributions library, and a subroutine for creating FE input file and calculating the related FE model parameters. At the start, a seed input value is fed manually to the random number generator where a random number is generated then the distribution library is called to calculate a value for a random variable based on a prescribed distribution type. Inverse transformation of the cumulative distribution function (CDF) of the random variable is used to obtain the random value for the variable based on the generated random number. The resulting random value for the variable is then saved. The seed input value is updated automatically based on the internally produced new seed in the random number generator. The previous operation is repeated to generate all random variables under consideration. The second step calls the FE input file subroutine designed specifically for a selected FE package. The subroutine takes the generated random variables from the previous step for the structural geometry, loads, and strength parameters to reproduce the FE input file according to the format required by the FE package.

## 2.6. Post Processing of Output

The post processing of the FE output files is the step that follows running the FE package in each simulation cycle as shown in Fig. 10. Subroutines were developed to conduct the post processing on a machine level. The first step is to read the FE output files and extract the values of interest. The extracted values are manipulated and used in failure recognition, then structural strength is evaluated. The probability of failure is then estimated, for example based on the conditional expectation variance reduction technique as described in Section 2.5.2. This technique requires load definition in the form of a probability distribution in order to use the strength-load reliability model shown in Fig. 11. The use of this model is also described in Section 2.5.2. Finally, relevant statistical measures for the failure probability are calculated. The statistical measures include the average failure probability, the coefficient of variation of failure probability, and the coefficient of variation of the sample average failure probability.

### 2.6.2. Reliability Assessment

Commonly used structural reliability assessment methods can be classified into two types, moment and simulation methods.

The moment methods were studied and described by many researchers (e.g., Ang and Tang 1984; Ayyub and Haldar 1984; Grigoriu 1982; Hasofer and Lind 1974; Melchers 1987; Shinozuka 1983; Thoft-Christensen and Baker 1982; and White and Ayyub 1985). In these methods, approximations are made about the distribution types, linearity of the failure surface, design or failure points, statistical characteristics of the basic random variables, and other parameters. Some of these methods are based on step-by-step approximations of the previous parameters in an optimization scheme and, consequently, lead to an improved estimate of the structure's reliability or probability of failure. However, such methods can have problems in convergence due to limitations in the level of nonlinearity of the failure surface that can be considered by the methods, the number of random variables that can be considered in the performance functions defining the potential failure modes of the structure, and the level of skewness of the probability distributions of the basic random variables.

In structural reliability, simulation-based methods determine the probability of failure of a structural component or system according to a specified performance function. They require complete information about the probabilistic characteristics of the basic random variables. In the classical use of the simulation-based methods, all the basic random variables are randomly generated and a performance equation for a failure mode is evaluated. Failures are then counted depending on the outcome of the evaluation. The probability of failure is estimated as the ratio of the number of failures to the total number of simulation cycles. Therefore, the smaller the probability of failure, the larger the needed number of simulation cycles to estimate the probability of failure with an acceptable level of statistical error. The efficiency of simulation can largely be improved by using variance reduction techniques. The result is a reduced level of computational effort and an increased analytical level. For example, Ayyub and Haldar (1984) and White and Ayyub (1985) suggested using conditional expectation and antithetic variates variance reduction techniques for structural reliability assessment. This method was determined to be efficient, and converges to the correct probability of failure in a relatively small number of simulation cycles. Other methods are based on common random numbers, importance sampling, and antithetic variates variance reduction techniques. Simulation methods are commonly used by researchers to validate other (non-simulation) methods and are generally considered accurate.

The main shortcoming of simulation methods is a large computational effort requirement for their use. This shortcoming is valid for the direct (hit or miss) method but not necessarily true for simulation with variance reduction techniques or selective sampling algorithms.

In this study, simulation with conditional expectation as a variance reduction technique is used for reliability assessment (Ayyub and Haldar 1984). The non-closed nature of the predicted strength based on finite element ($B$) does not allow for choosing any basic variable $X_i$ in $B$ as the conditioned random variable even if $X_i$ is the one with the highest variability level. Because $B$ has to be computed using a non-closed form based on FE methods. The only remaining choices are the load variables, for example, stillwater lead ($Q_s$) or wave load ($Q_w$). If $Q_w$ has a higher variability level than $Q_s$, then the survival probability $P_s$ is given by

$$\begin{aligned} P_s &= P(Y > 0) \\ &= P(B - Q_s - Q_w > 0) \\ &= P(Q_w < B - Q_s) \\ &= E\left[F_{Q_w}(B - Q_s)\right] \end{aligned} \tag{2-1}$$

where $Y$ = performance function; $B = B(X_1, X_2, \ldots, X_{n-2})$ as the strength variable that is a function of $n$-2 basic random variables, $Q_w$ = the cumulative distribution function of $Q_w$, and $E(.)$ is the expected value. Therefore, the failure probability $P_f$ can be determined as

$$P_f = 1 - P_s = 1 - E\left[F_{Q_w}(B - Q_s)\right] \tag{2-2}$$

Therefore, each simulation cycle ($i\underline{th}$ cycle) is expected to produce a failure probability $P_{fi}$ based on evaluating the cumulative distribution function of $Q_w$ at generated values of $B$ and $Q_s$ (i.e., $B_i$ and $Q_{si}$). The sample mean of the probability of failure ($\overline{P}_f$) is then computed as

$$\overline{P}_f = \frac{1}{N}\left(\sum_{i=1}^{N} P_{fi}\right) \tag{2-3}$$

in which $N$ = the number of simulation cycles. This estimate of $P_f$ can be considered an unbiased estimator of the population value. The variance associated with this estimated value is (Ang and Tang 1975)

$$Var(\overline{P}_f) = \frac{Var(P_f)}{N} = \frac{1}{N}\left(\frac{1}{N-1}\sum_{i=1}^{N}(P_{fi} - \overline{P}_f)\right) \tag{2-4a}$$

where $Var(\overline{P}_f)$ indicates the accuracy in estimating $\overline{P}_f$. A smaller value of $Var(\overline{P}_f)$ is always preferred. The coefficient of variation (COV) for the estimated failure probability is given by

$$\mathrm{COV}(\overline{P}_f) = \frac{\sqrt{\frac{1}{N}\left(\frac{1}{N-1}\sum_{i=1}^{N}(P_{fi} - \overline{P}_f)\right)}}{\overline{P}_f} \tag{2-4b}$$

## 3. EXAMPLE

The example presented in this section illustrates the application of the methodology to a stiffened shell-type structure. A small-scale stiffened panel of a ship structure tested by Faulkner (1977) is used in this example. The panel was also investigated by Hess, et al (1994). The panel was modeled as a simply-supported structure with two states of loading: (1) concentric axial compression only, and (2) concentric axial compression with lateral pressure. The primary failure mode for this panel is an elastic non-linear instability. A commercially-available FE package was selected based on its ability to deal with this failure mode. The selected package, ABAQUS, was used to predict the buckling eigenvalues.

### 3.1. Geometry and Material

The panel consists of a rectangular thin plate stiffened with equally spaced parallel T-shaped stiffeners as shown in Fig. 12. The overall plate dimensions are 854 × 790 mm with a 3.0 mm plate thickness. The web height is 26.66 mm with a web thickness of 4.9 mm. The flange width is 25.4 mm with a flange thickness of 5.84 mm. The five equally-spaced stiffeners are 183-mm apart, they are parallel to the short side of the plate and the plate extends equally beyond the external stiffeners a distance equal to 61 mm each way. The material is assumed to be isotropic elastic with a modulus of elasticity E = 208000 MPa and Poisson's ratio $\nu = 0.3$.

### 3.2. Loading and Boundary Conditions

Two loading conditions were considered: (1) the panel was loaded in the short direction by a concentric uniformly distributed edge load, and (2) the panel was simultaneously loaded in the short direction by concentric uniformly distributed edge loads and a lateral pressure on the plate surface. The lateral pressure values of -0.07, -0.035, 0.0, 0.035, and 0.07 MPa were taken (where a positive pressure is normal to the plate creating compressive bending stress in the plate). The concentric, uniformly-distributed edge loads were thickness dependent to account for the possible variability in the thickness. The panel was simply supported in the plate level on two opposite edge, namely, L01 and L03.

### 3.3. FE model

For the purpose of FE modeling, the middle planes of the plate, webs, and flanges were considered reference planes for dimensioning. Consequently, the considered dimensions of the FE model were 854 × 970 × 3 mm for the plate, 31.08 × 4.9 mm for the web and 25.4 × 5.84 mm for the flange. The model was based on the 8-node doubly-curved shell element S8R5, which could provide the eigenvalue buckling estimates. In general, shell buckling stability requires two types of analysis. First, eigenvalue analysis was used to obtain estimates of the buckling loads and modes. This type of analysis provides guidance in mesh design to ensure the convergence of eigenvalue estimates of buckling loads. The mesh needs to be adequate to model the buckling modes which are usually more complex than the prebuckling deformation mode. The second phase of the study is to perform load-displacement analyses.

The key aspect of the eigenvalue analysis is the mesh design. For the example under study, the buckling could be an overall buckling of the whole panel or a local buckling in either the

plate, the web, or the flange. In this demonstration, the whole panel was modeled without accounting to any possible symmetry. The following meshes were used: $6 \times 10$ for the plate, $1 \times 6$ for each web, and $2 \times 6$ for each flange as shown in Fig. 4.

### 3.4. Random Variables

The basic geometric and material random variables were defined based on reported values, practical aspects, and judgment (Ayyub, et al 1994, Ellingwood, et al 1980). The adopted values are summarized in Table 1 and detailed in Tables 2 through 8. The overall number of random variables that were considered in this example is 55. They included random variables at the three levels, i.e., the plate, web, and flange that are described in section 2.4. Six geometric variables were defined at the plate level, 25 variables at the web level, and 20 variables at the flange level. In addition, three variables were defined for thickness one for each level and the modulus of elasticity was assumed to be the same for the three levels and expressed by one random variable. The random variables were divided into two categories. A random variable in the first category was defined by its mean values and standard deviations whereas a random variable in the second category was defined by its mean values and coefficients of variation (COV). The standard deviation for the overall plate size was assumed to be 4.0 mm and for its out-of-plane distortion was assumed to be 1.0 mm. The web tilting, bowing, and flange tilting have standard deviations of 0.5 mm, 0.1 mm, and 0.2 mm respectively. The variability of the thickness at all levels was expressed by 4% COV, and the web height and the flange width by 2.5% COV. The COV of the modulus of elasticity was taken as 4%. A normal probability distribution function (PDF) was used for all basic random variables.

### 3.5. Strength and Reliability Assessment

The panel strength prediction and reliability assessments were based on generated values of the basic random variables and those of the applied load. The axial strength (buckling load) for the panel was evaluated by manipulating the estimated eigenvalues obtained by the FE analysis. The load was assumed to have a lognormal distribution with an assumed mean value two thirds of the nominal buckling strength and COV of 10%. Reliability assessment was conducted using simulation with conditional expectation variance reduction technique as described in Section 2.5.2. For the purpose of demonstration, conditional expectation as defined in Equations. 2-1 through 2-4 was used with one loading type. The cumulative distribution function of the load (lognormal distribution) was used to determine the failure probability in each simulation cycle.

#### 3.5.1. Concentric Axial Compression Loading

Two cases are discussed in this section, the nominal case which does not account for any type of variability and the reference case which is defined according to the data in Tables 2 through 8.

##### 3.5.1.1. Nominal Case

Panel strength Prediction for this case was based on the nominal values of all variables without accounting for any type of variability. The panel was loaded in the short direction (parallel to the stiffeners) by a uniformly distributed concentric compressive load acting on both edges, and simply supported at the plate level in the short direction. The estimated axial strength (buckling load) was determined to be 273.3 MPa, the failure probability based on random

loading was assessed to be $1.51\times10^{-5}$, and the predicted failure mode was a local buckling in the plate as shown in Fig. 13. The same panel was investigated by Hess, et al (1994) using several combinations of concentric axial and transverse in-plane loads, and lateral pressure. The model was simply supported at the elastic centroid. Strength statistics were estimated using Monte Carlo simulation. The panel axial strength was predicted using an algorithm as defined by Hughes (1988). The examples presented by Hess et al (1994) were analyzed with an assumed eccentricity. The predicted mean axial strengths for the case of zero lateral pressure and zero transverse loads, which corresponds to the nominal case, but with an eccentricities of −0.5273 mm and +0.5273 mm were 247.5 MPa and 156.5 MPa, respectively. The differences in the axial strength values between the presented work herein and that of Hess et al (1994) can be attributed to the presence of the eccentricity and the difference in boundary conditions.

#### 3.5.1.2. Reference Case

The reference case accounts for geometric and material variability as defined in Tables 2 through 8. The axial strength and failure probability results are shown in Figs. 14 and 15 respectively. Figure 14 shows the histograms and statistical measures for both axial strength and a normalized axial strength with respect to the nominal case based on 500 simulation cycles. The average axial strength value was predicted to be 273.9 MPa. Figure 15 shows the convergence of the average failure probability as the number of simulation cycles is increased and the statistical measures of the failure probability with its histogram using 500 simulation cycles.

### 3.5.2. Concentric Axial Compression Loading With Lateral Pressure

The axial strength of the panel under concentric axial compression loading and lateral pressure was predicted for both the nominal and reference cases. The lateral pressure was varied from 0.07 MPa to −0.07 Mpa (a positive pressure is in the positive direction of a normal vector to the plate, i.e., its direction from the plate to the stiffeners). The mean strength, nominal strength, and the mean strength plus and minus one and two standard deviations are shown in Fig. 16 for the whole range of the lateral pressure from −0.07 to +0.07 Mpa. The strength decreases with the increase in pressure which is in agreement with the predicted failure mode. For a local buckling failure mode for the plate, applying a positive pressure means an increase in the compressive stresses in the plate, consequently a decrease in the axial strength of the panel.

## 3.6. Parametric Analysis

A parametric analysis was conducted for the axial strength and failure probability of the panel. The analysis was carried out by individually varying the coefficients of variation or standard deviations of the basic random variables. The notations, mean values, and ranges of COV and standard deviations of the random variables are given in Table 9. The following observations were developed based on the results of the parametric analysis using 100 simulation cycles:

- For the plate width, Fig. 17 shows that increasing the COV from 0.47% to 0.94%, the normalized strength decreases from 1.007 to 0.988, the COV of the axial strength decreases from 8.87% to 7.79%, and the average of probability of failure decreases from $8.20\times10^{-4}$ to $6.45\times10^{-4}$.

- For the plate out-of-plane distortion, Fig. 18 shows that increasing the standard deviation from 1.0 to 3.0, the normalized strength increases from 1.007 to 1.009, the COV of the axial strength decreases from 8.87% to 7.42%, and the average of probability of failure decreases from $8.20\times10^{-4}$ to $6.45\times10^{-4}$.
- For the web height, Fig. 19 shows that increasing the COV from 2.5% to 5.0%, the normalized strength increases from 1.007 to 1.011, the COV of the axial strength decreases from 8.87% to 7.37%, and the average of probability of failure decreases from $8.20\times10^{-4}$ to $3.94\times10^{-4}$.
- For the web tilting, Fig. 20 shows that increasing the standard deviation from 0.2 mm to 0.5 mm, the normalized strength increases from 1.005 to 1.007, the COV of the axial strength increases from 7.17% to 9%, and the average of probability of failure increases from $5.0\times10^{-4}$ to $8.45\times10^{-4}$.
- For the web bowing, Fig. 21 shows that increasing the standard deviation from 0.1 mm to 0.2 mm, the normalized strength decreases from 1.007 to 0.99, the COV of the axial strength decreases from 9.0% to 7.8%, and the average of probability of failure decreases from $8.2\times10^{-4}$ to $4.84\times10^{-4}$.
- For the flange width, Fig. 22 shows that increasing the COV from 2.5% to 5.0%, the normalized strength decreases from 1.007 to 1.004, the COV of the axial strength decreases from 9.0% to 7.26%, and the average of probability of failure decreases from $8.2\times10^{-4}$ to $2.23\times10^{-4}$.
- For the flange tilting, Fig. 23 shows that increasing the standard deviation from 0.2 mm to 0.5 mm, the normalized strength decreases from 1.007 to 0.995, the COV of the axial strength decreases from 9.0% to 8.0%, and the average of failure probability increases from $8.2\times10^{-4}$ to $1.77\times10^{-3}$.
- For the thicknesses, Fig. 24 shows that increasing the COV from 4.0% to 8.0%, the normalized strength decreases from 1.007 to 0.994, the COV of the axial strength increases from 8.87% to 13.0%, and the average of probability of failure decreases from $8.28\times10^{-4}$ to $1.53\times10^{-2}$.
- For the modulus of elasticity, Fig. 24 shows that increasing the COV from 4.0% to 8.0%, the normalized strength decreases from 1.007 to 1.002, the COV of the axial strength remains constant at the value of 8.90%, and the average of probability of failure decreases from $8.2\times10^{-4}$ to $1.49\times10^{-3}$.

The above failure probability observations were based on results from 100 simulation cycles. The number of simulation cycles might not be adequate for obtaining accurate failure probability results, but it is sufficient for determining the axial strength. The number of cycles was limited to 100 in order make the study feasible within the planned time frame of the project.

Table 10 shows a summary of the results of the parametric study. According to the table, variations in the variability of plate size and web bowing produced the largest effect on the mean axial strength ratio; whereas variations in the variability of thickness of the plate, webs, and flanges produced the largest effect on the coefficient of variation of the axial strength.

# 4. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS FOR FUTURE WORK

## 4.1. Summary and Conclusions

A prototype computational methodology for reliability assessment of continuum structures using finite element analysis with instability failure modes was described in this report. Examples were used to illustrate and test the methodology. Geometric and material uncertainties were considered in the finite element model. A computer program was developed to implement this methodology by integrating uncertainty formulations to create a finite element input file and to conduct the reliability assessment on a machine level. A commercial finite element package was used as a basis for the strength assessment in the presented procedure. A parametric study for a stiffened panel strength was also carried out. The following observations were made based on the parametric study:

- Increasing the COV of the plate width resulted in a slight decrease of the normalized strength, and its COV.
- Increasing the standard deviation of the plate out-of-plane distortion resulted in a slight increase of the normalized strength and a moderate decrease in its COV.
- Increasing the COV of the web height resulted in a moderate increase of the normalized strength and a moderate decrease of its COV.
- Increasing the standard deviation of the web tilting resulted in a slight increase of the normalized strength and a significant increase of its COV.
- Increasing the standard deviation of the web bowing resulted in a moderate decrease of the normalized strength and a moderate decrease of its COV.
- Increasing the COV of the flange width resulted in a slight decrease of the normalized strength and a moderate decrease of its COV.
- Increasing the standard deviation of the flange tilting resulted in a moderate decrease of the normalized strength and a slight decrease in its COV.
- Increasing the COV of the thickness resulted in a moderate decrease of the normalized strength and a significant increase of its COV.
- Increasing the COV of the modulus of elasticity resulted in a slight decrease of the normalized strength and no change for its COV.
- It can be concluded that variations in the variability of plate size and web bowing produced the largest effect on the mean axial strength ratio whereas variations in the variability of thickness of the plate, webs, and flanges produced the largest effect on the coefficient of variation of the axial strength.

## 4.2. Recommendations for Future Work

Based on this study, for future work we recommend:

- The feasibility of using the developed method for complex structures with multiple failure modes needs to be investigated. The structures need to be selected such that methods for failure recognition and classification as demonstrated in Figs. 1 and 2 can be developed.
- The effects of failure recognition and classification for continuum structures on reliability estimates need to studied.

## GLOSSARY

The following symbols are used in this paper:

| | | |
|---|---|---|
| $B$ | = | predicted strength |
| CDF | = | cumulative distribution function |
| COV | = | coefficient of variation |
| $F$ | = | cumulative distribution function |
| L0i | = | plate side dimensions |
| L1i | = | web side dimensions |
| L2i | = | flange side dimensions |
| L11i | = | web height, side 1 |
| L13i | = | web height, side 3 |
| L21i | = | flange width, side 1 |
| L23i | = | flange width, side 3 |
| $N$ | = | number of simulation cycles |
| PDF | = | probability density function |
| $Q_s$ | = | still water load |
| $Q_w$ | = | wave load |
| $P_f$ | = | failure probability |
| $\overline{P}_f$ | = | sample mean of failure probability |
| $P_s$ | = | survival probability |
| t0 | = | plate thickness |
| t1 | = | web thickness |
| t2 | = | flange thickness |
| $Var$ | = | Variance |
| X21i | = | web tilting, side 1 |
| X23i | = | web tilting, side 3 |
| $Y$ | = | performance function |
| zP0i | = | plate corners coordinates |
| Z2i0 | = | flange tilting, side y = 0 |
| Z2iL | = | flange tilting, side y = L |
| XBOi | = | web bowing |

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Table 1. Geometric And Material Random Variables for the Stiffened Panel

| Variable no | Geometrical variables | Notation | Mean value | Coefficient of variation (COV) | Standard deviation |
|---|---|---|---|---|---|
| 1 | Plate size (mm) | L0i | 854 | | 4.0 |
| 2 | Plate thickness (mm) | $t_0$ | 3.0 | 4% | 0.12 |
| 3 | Web thickness (mm) | $t_1$ | 4.9 | 4% | 0.196 |
| 4 | Flange thickness (mm) | $t_2$ | 5.84 | 4% | 0.234 |
| 5 | Plate-out of plane distortion (mm) | zP0i | 0.0 | | 1.0 |
| 6 | Web height (mm) | L1ji | 31.08 | 2.5% | 0.77 |
| 7 | Web tilting (mm) | X2ji | 0.0 | | 0.5 |
| 8 | Web bowing (mm) | XB0i | 0.0 | | 0.1 |
| 9 | Flange width (mm) | L2ji | 25.4 | 2.5% | 0.635 |
| 10 | Flange tilting (mm) | Z2i0, Z2iL | 0.0 | | 0.2 |
| 11 | Modulus of elasticity (MPa) | E | 208000 | 4% | 8320 |
| 12 | Poisson's ratio | $\nu$ | | | |
| 13 | Yield stress (KPa)[1] | $F_y$ | 250000 | 7% | 17500 |

[1] Nominal yield stress = 240000 kPa

Table 2. Thickness and Plate Geometric Variables

| Variable no. | | Geometrical variables | Notation | Mean value (mm) | Coefficient of variation (COV) | Standard deviation (mm) |
|---|---|---|---|---|---|---|
| global | local | | | | | |
| 1 | 1 | Panel width (side 1) | $L_{01}$ | 854.0 | | 4.0 |
| 2 | 2 | Panel width (side 3) | $L_{03}$ | 854.0 | | 4.0 |
| | | | | | | |
| 3 | 3 | Plate thickness | $t_p$ | 3.0 | 4% | 0.12 |
| 4 | 4 | Web thickness | $t_w$ | 4.9 | 4% | 0.196 |
| 5 | 5 | Flange thickness | $t_f$ | 5.84 | 4% | 0.234 |
| | | | | | | |
| 6 | 6 | Plate-out of plane distortion (corner2) | $Z_{P02}$ | 0.0 | | 1.0 |
| 7 | 7 | Plate-out of plane distortion (corner3) | $Z_{P03}$ | 0.0 | | 1.0 |
| 8 | 8 | Plate-out of plane distortion (corner4) | $Z_{P04}$ | 0.0 | | 1.0 |

Table 3. Web Height Variables

| Variable no. | | Geometrical variables | Notation | Mean value (mm) | Coefficient of variation (COV) | Standard deviation (mm) |
|---|---|---|---|---|---|---|
| global | local | | | | | |
| 9 | 1 | Height of web no. 1 (side 1) | $L_{111}$ | 31.08 | 2.5% | 0.77 |
| 10 | 2 | Height of web no. 2 (side 1) | $L_{112}$ | 31.08 | 2.5% | 0.77 |
| 11 | 3 | Height of web no. 3 (side 1) | $L_{113}$ | 31.08 | 2.5% | 0.77 |
| 13 | 4 | Height of web no. 4 (side 1) | $L_{114}$ | 31.08 | 2.5% | 0.77 |
| 14 | 5 | Height of web no. 5 (side 1) | $L_{115}$ | 31.08 | 2.5% | 0.77 |
| | | | | | | |
| 15 | 6 | Height of web no.1 (side 3) | $L_{131}$ | 31.08 | 2.5% | 0.77 |
| 16 | 7 | Height of web no.2 (side 3) | $L_{132}$ | 31.08 | 2.5% | 0.77 |
| 17 | 8 | Height of web no.3 (side 3) | $L_{133}$ | 31.08 | 2.5% | 0.77 |
| 18 | 9 | Height of web no.4 (side 3) | $L_{134}$ | 31.08 | 2.5% | 0.77 |
| 19 | 10 | Height of web no.5 (side 3) | $L_{135}$ | 31.08 | 2.5% | 0.77 |

Table 4. Web Tilting Variables

| Variable no. | | Geometrical variables | Notation | Mean value (mm) | Coefficient of variation (COV) | Standard deviation (mm) |
|---|---|---|---|---|---|---|
| global | local | | | | | |
| 20 | 1 | Tilting of web no.1 (side 1) | $X_{211}$ | 0.0 | | 0.5 |
| 21 | 2 | Tilting of web no.2 (side 1) | $X_{212}$ | 0.0 | | 0.5 |
| 22 | 3 | Tilting of web no.3 (side 1) | $X_{213}$ | 0.0 | | 0.5 |
| 23 | 4 | Tilting of web no.4 (side 1) | $X_{214}$ | 0.0 | | 0.5 |
| 24 | 5 | Tilting of web no.5 (side 1) | $X_{215}$ | 0.0 | | 0.5 |
| | | | | | | |
| 25 | 6 | Tilting of web no.1 (side 3) | $X_{231}$ | 0.0 | | 0.5 |
| 26 | 7 | Tilting of web no.2 (side 3) | $X_{232}$ | 0.0 | | 0.5 |
| 27 | 8 | Tilting of web no.3 (side 3) | $X_{233}$ | 0.0 | | 0.5 |
| 28 | 9 | Tilting of web no.4 (side 3) | $X_{234}$ | 0.0 | | 0.5 |
| 29 | 10 | Tilting of web no.5 (side 3) | $X_{235}$ | 0.0 | | 0.5 |

Table 5. Web Bowing Variables

| Variable no. | | Geometrical variables | Notation | Mean value (mm) | Coefficient of variation (COV) | Standard deviation (mm) |
|---|---|---|---|---|---|---|
| global | local | | | | | |
| 45 | 1 | Bowing of web no. 1 (side 3) | $X_{B01}$ | 0.0 | | 0.1 |
| 46 | 2 | Bowing of web no. 2 (side 3) | $X_{B02}$ | 0.0 | | 0.1 |
| 47 | 3 | Bowing of web no. 3 (side 3) | $X_{B03}$ | 0.0 | | 0.1 |
| 48 | 4 | Bowing of web no. 4 (side 3) | $X_{B04}$ | 0.0 | | 0.1 |
| 49 | 5 | Bowing of web no. 5 (side 3) | $X_{B05}$ | 0.0 | | 0.1 |

Table 6. Flange Width Variables

| Variable no. | | Geometrical variables | Notation | Mean value (mm) | Coefficient of variation (COV) | Standard deviation (mm) |
|---|---|---|---|---|---|---|
| global | local | | | | | |
| 30 | 1 | Width of flange no. 1 (side 1) | $L_{211}$ | 25.4 | 2.5% | 0.635 |
| 31 | 2 | Width of flange no. 2 ( side 1) | $L_{212}$ | 25.4 | 2.5% | 0.635 |
| 32 | 3 | Width of flange no.3 ( side 1) | $L_{213}$ | 25.4 | 2.5% | 0.635 |
| 33 | 4 | Width of flange no. 4 ( side 1) | $L_{214}$ | 25.4 | 2.5% | 0.635 |
| 34 | 5 | Width of flange no. 5 ( side 1) | $L_{215}$ | 25.4 | 2.5% | 0.635 |
| | | | | | | |
| 35 | 6 | Width of flange no.1 ( side 3) | $L_{231}$ | 25.4 | 2.5% | 0.635 |
| 36 | 7 | Width of flange no.2 ( side 3) | $L_{232}$ | 25.4 | 2.5% | 0.635 |
| 37 | 8 | Width of flange no.3 ( side 3) | $L_{233}$ | 25.4 | 2.5% | 0.635 |
| 38 | 9 | Width of flange no.4 ( side 3) | $L_{234}$ | 25.4 | 2.5% | 0.635 |
| 39 | 10 | Width of flange no.5 ( side 3) | $L_{235}$ | 25.4 | 2.5% | 0.635 |

Table 7. Flange Tilting Variables

| Variable no. | | Geometrical variables | Notation | Mean value (mm) | Coefficient of variation (COV) | Standard deviation (mm) |
|---|---|---|---|---|---|---|
| global | local | | | | | |
| 40 | 1 | Tilting of flange no.1 (side 1) | $Z_{210}$ | 0.0 | | 0.2 |
| 41 | 2 | Tilting of flange no.2 (side 1) | $Z_{220}$ | 0.0 | | 0.2 |
| 42 | 3 | Tilting of flange no.3 (side 1) | $Z_{230}$ | 0.0 | | 0.2 |
| 43 | 4 | Tilting of flange no.4 (side 1) | $Z_{240}$ | 0.0 | | 0.2 |
| 44 | 5 | Tilting of flange no.5 (side 1) | $Z_{250}$ | 0.0 | | 0.2 |
| | | | | | | |
| 45 | 6 | Tilting of flange no.1 (side 3) | $Z_{21L}$ | 0.0 | | 0.2 |
| 46 | 7 | Tilting of flange no.2 (side 3) | $Z_{22L}$ | 0.0 | | 0.2 |
| 47 | 8 | Tilting of flange no.3 (side 3) | $Z_{23L}$ | 0.0 | | 0.2 |
| 48 | 9 | Tilting of flange no.4 (side 3) | $Z_{24L}$ | 0.0 | | 0.2 |
| 49 | 10 | Tilting of flange no.5 (side 3) | $Z_{25L}$ | 0.0 | | 0.2 |

Table 8. Material Variability

| Variable no. | | Material variables | Notation | Mean value | Coefficient of variation (COV) | Standard deviation |
|---|---|---|---|---|---|---|
| global | local | | | | | |
| 50 | 1 | Modulus of elasticity of plate material (MPa) | $E_0$ | 208000 | 4% | 8320 |
| 51 | 2 | Modulus of elasticity of web material (MPa) | $E_1$ | 208000 | 4% | 8320 |
| 52 | 3 | Modulus of elasticity of flange material (MPa) | $E_2$ | 208000 | 4% | 8320 |
| | | | | | | |
| 53 | 4 | Poisson's ratio of plate | $\nu_0$ | | | |
| 54 | 5 | Poisson's ratio of web | $\nu_1$ | | | |
| 55 | 6 | Poisson's ratio of flange | $\nu_2$ | | | |
| | | | | | | |
| 56 | 7 | Yield stress of plate (kPa) [1] | $F_{y0}$ | 250000 | 7% | 17500 |
| 57 | 8 | Yield stress of web (kPa) [1] | $F_{y1}$ | 250000 | 7% | 17500 |
| 57 | 9 | Yield stress of flange (kPa) [1] | $F_{y2}$ | 250000 | 7% | 17500 |

[1] Nominal yield stress = 240000 kPa

Table 9. Variation of Coefficient of Variation or Standard Deviation

| Variable no. | Geometrical variables | Notation | Mean value | Coefficient of variation (COV) | Standard deviation |
|---|---|---|---|---|---|
| 1 | Plate size (mm) | L0i | 854 | | 4.0 to 8 |
| 2 | Plate thickness (mm) | $t_0$ | 3.0 | 4 to 8% | |
| 3 | Web thickness (mm) | $t_1$ | 4.9 | 4 to 8% | |
| 4 | Flange thickness (mm) | $t_2$ | 5.84 | 4 to 8% | |
| 5 | Plate-out of plane distortion (mm) | zP0i | 0.0 | | 1.0 to 3.0 |
| 6 | Web height (mm) | L1ji | 31.08 | 2.5 to 5% | |
| 7 | Web tilting (mm) | X2ji | 0.0 | | 0.2 to 0.5 |
| 8 | Web bowing (mm) | XB0i | 0.0 | | 0.1 to 0.2 |
| 9 | Flange width (mm) | L2ji | 25.4 | 2.5 to 5% | |
| 10 | Flange tilting (mm) | Z2i0, Z2iL | 0.0 | | 0.2 to 0.5 |
| 11 | Modulus of elasticity (MPa) | E | 208000 | 4 to 8% | |

Table 10. Parametric Analysis Results

| Variable no. | Geometrical Variables | Mean value | Variation of coefficient of variation | Variation of standard deviation | Effect on axial strength ratio | Effect on coefficient of variation of strength |
|---|---|---|---|---|---|---|
| 1 | Plate size (mm) | 854 | | 4.0 to 8.0 | High | Medium/Low |
| 2 | Plate thickness (mm) | 3.0 | 4 to 8% | 0.12 to 0.24 | Medium | High |
| 3 | Web thickness (mm) | 4.9 | 4 to 8% | 0.196 to 0.392 | Medium | High |
| 4 | Flange thickness (mm) | 5.84 | 4 to 8% | 0.234 to 0.468 | Medium | High |
| 5 | Plate-out of plane distortion (mm) | 0.0 | | 1.0 to 3.0 | Low | Medium/Low |
| 6 | Web height (mm) | 31.08 | 2.5 to 5% | 0.77 to 1.54 | Low | Medium/Low |
| 7 | Web tilting (mm) | 0.0 | | 0.2 to 0.5 | Low | Medium/Low |
| 8 | Web bowing (mm) | 0.0 | | 0.1 to 0.2 | High | Medium/Low |
| 9 | Flange width (mm) | 25.4 | 2.5 to 5% | 0.635 to 1.27 | Low | Medium/Low |
| 10 | Flange tilting (mm) | 0.0 | | 0.2 to 0.5 | Medium | Medium/Low |
| 11 | Modulus of elasticity (MPa) | 208000 | 4 to 8% | 8320 to 16640 | Low | None |

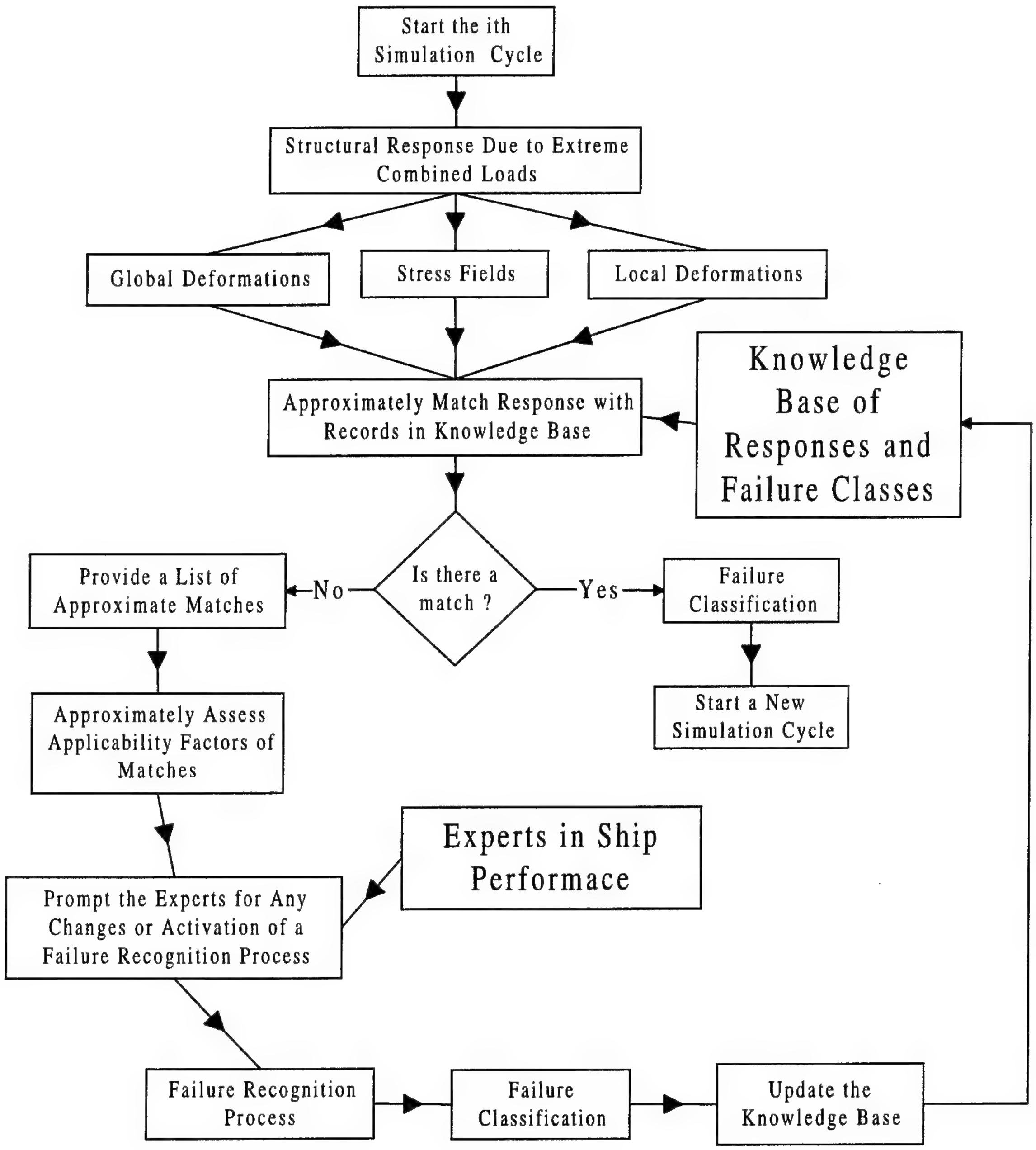



Figure 1. Failure Classification

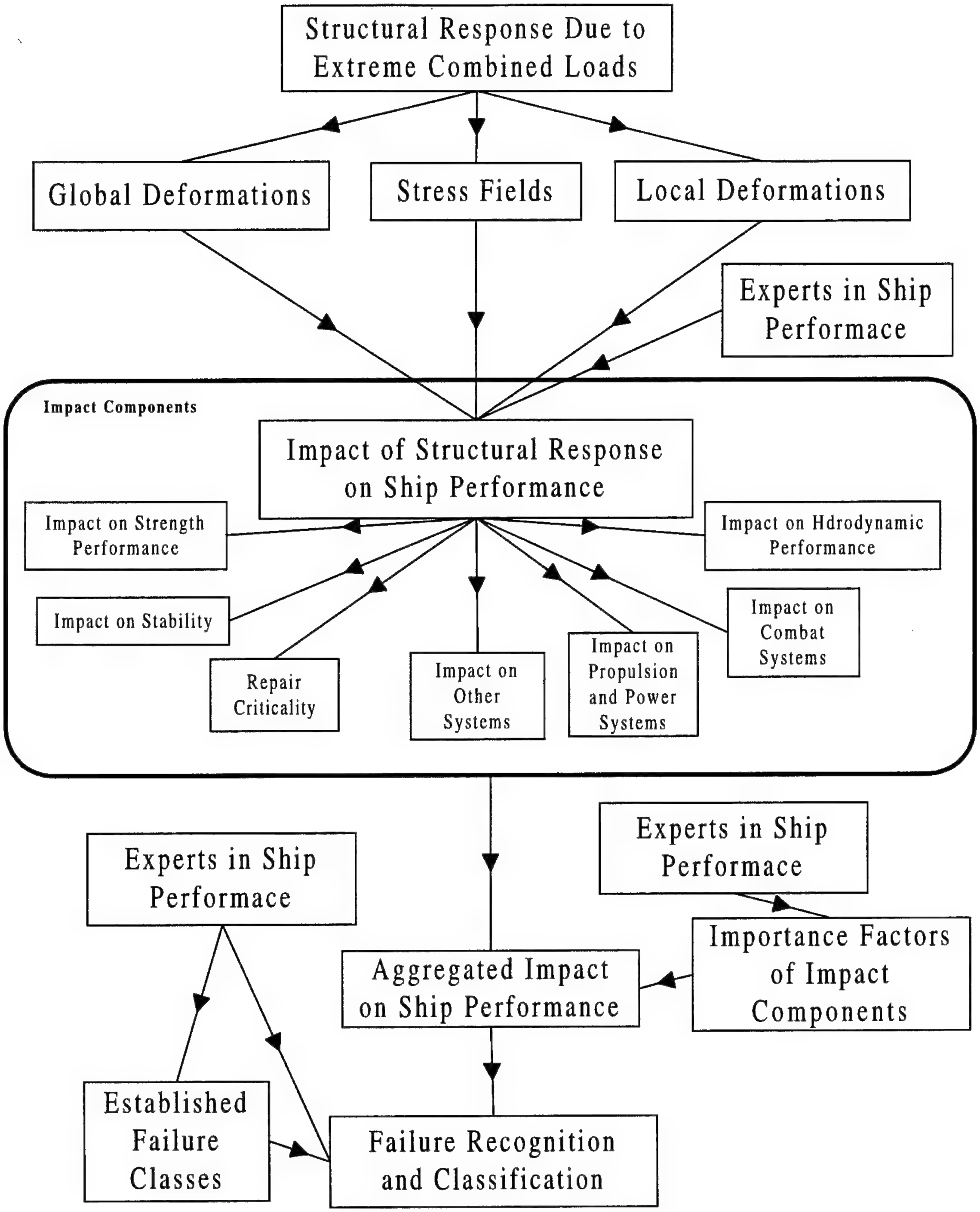
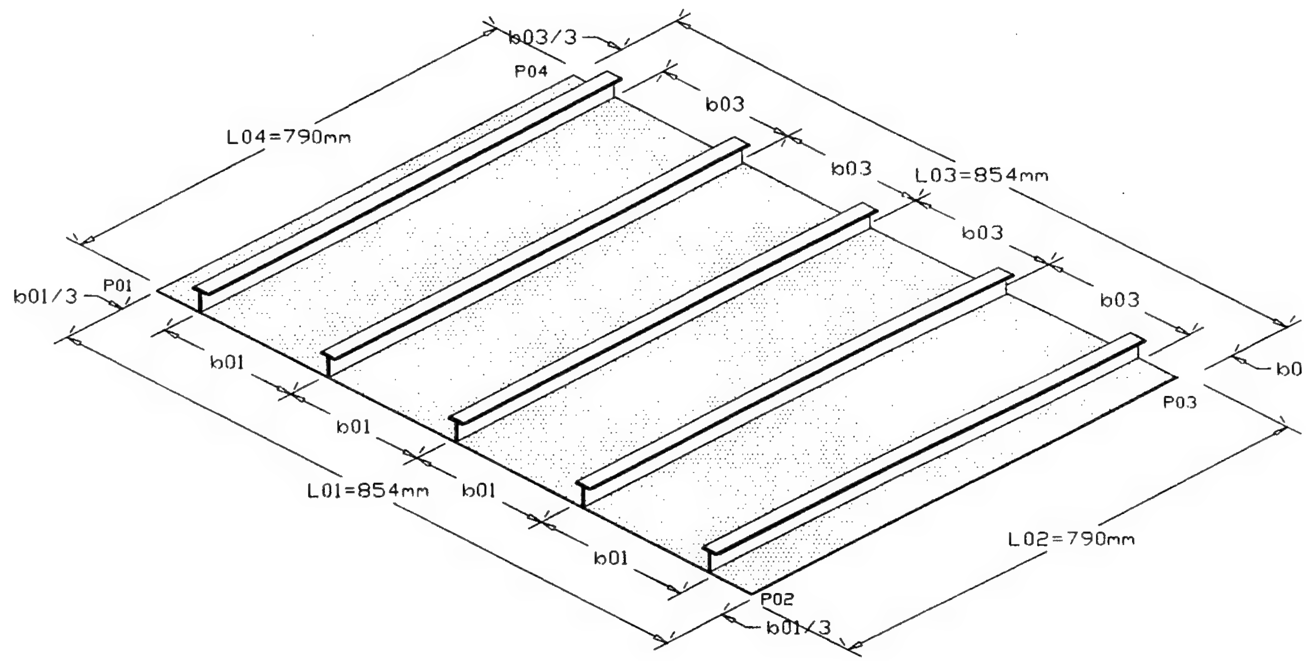



Figure 2. Failure Recognition and Classification




**STIFFENED PANEL**

Figure 3. Typical Stiffened Panel

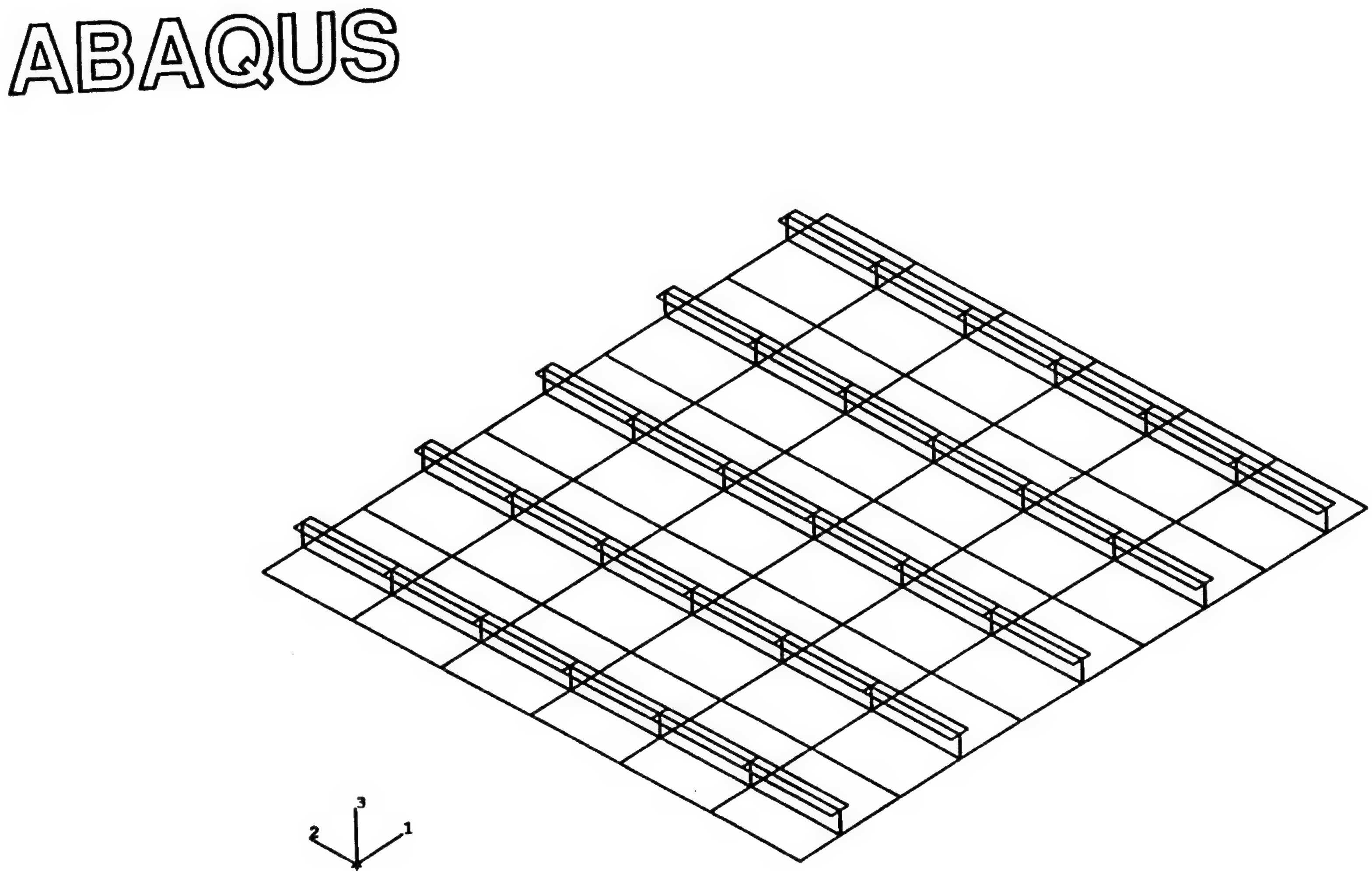



Figure 4. Finite Element Mesh of the Stiffened Panel

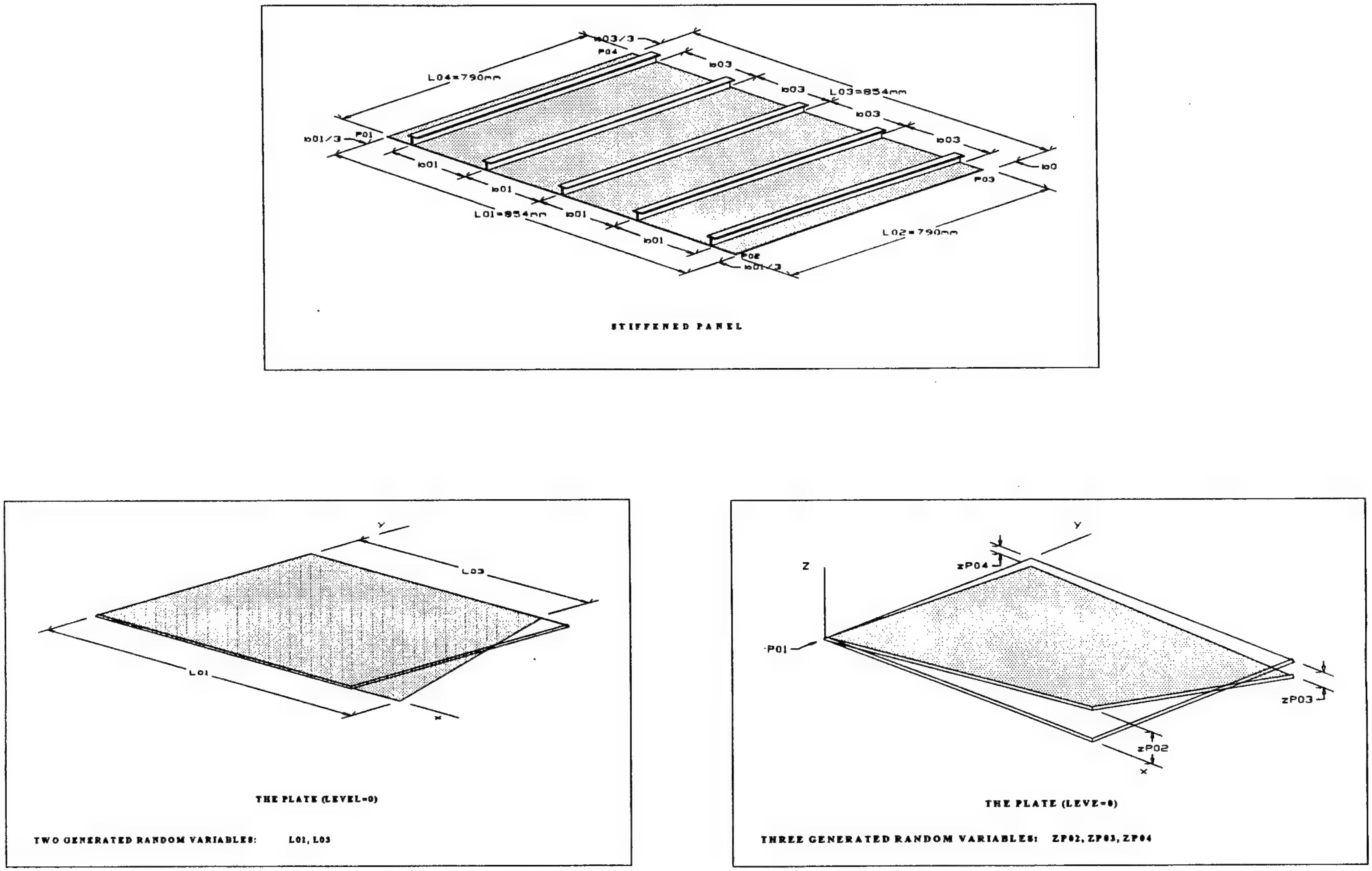



Figure 5. Plate Width Variability and Plate Distortion

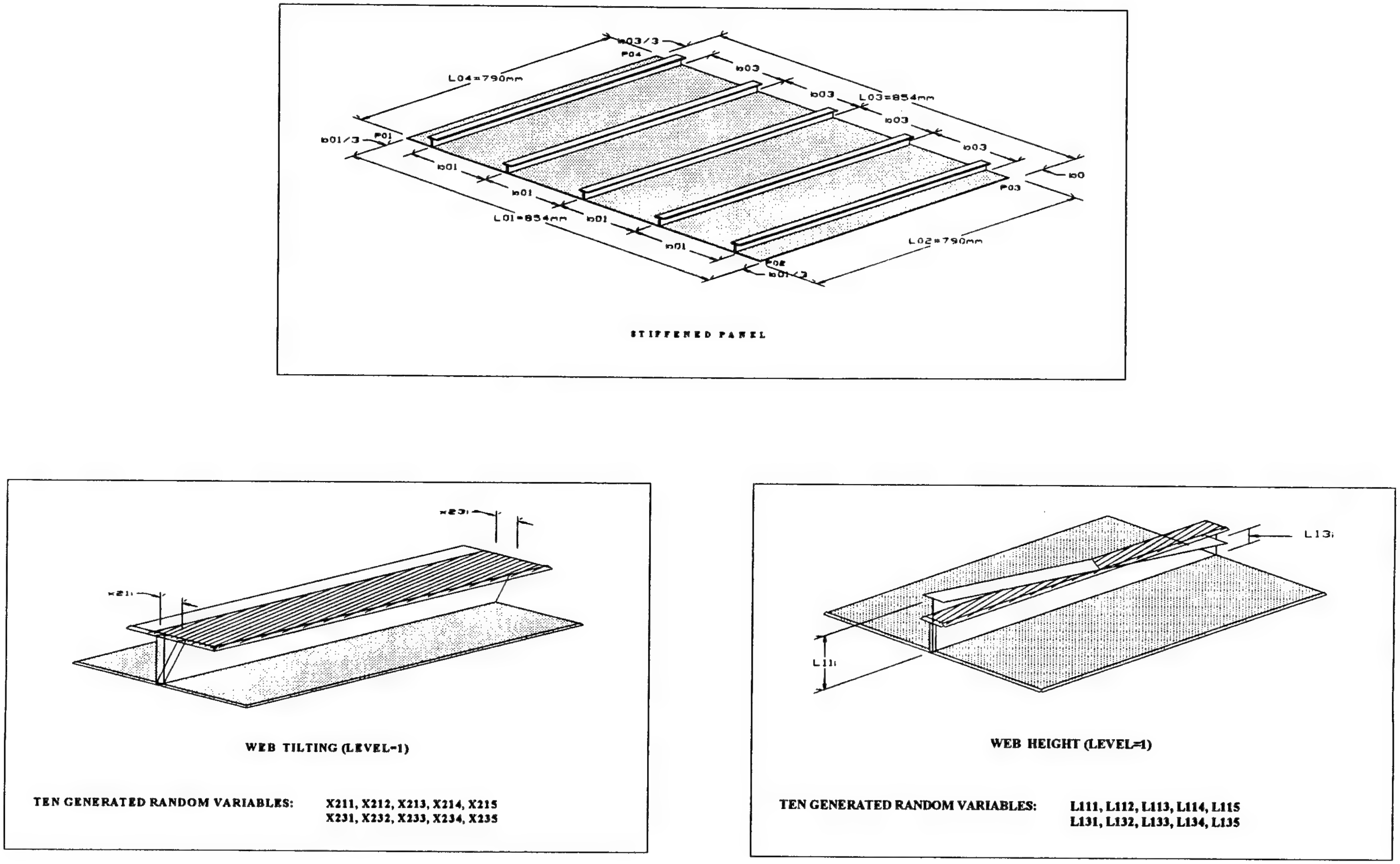



Figure 6. Web Height Variability and Web Tilting

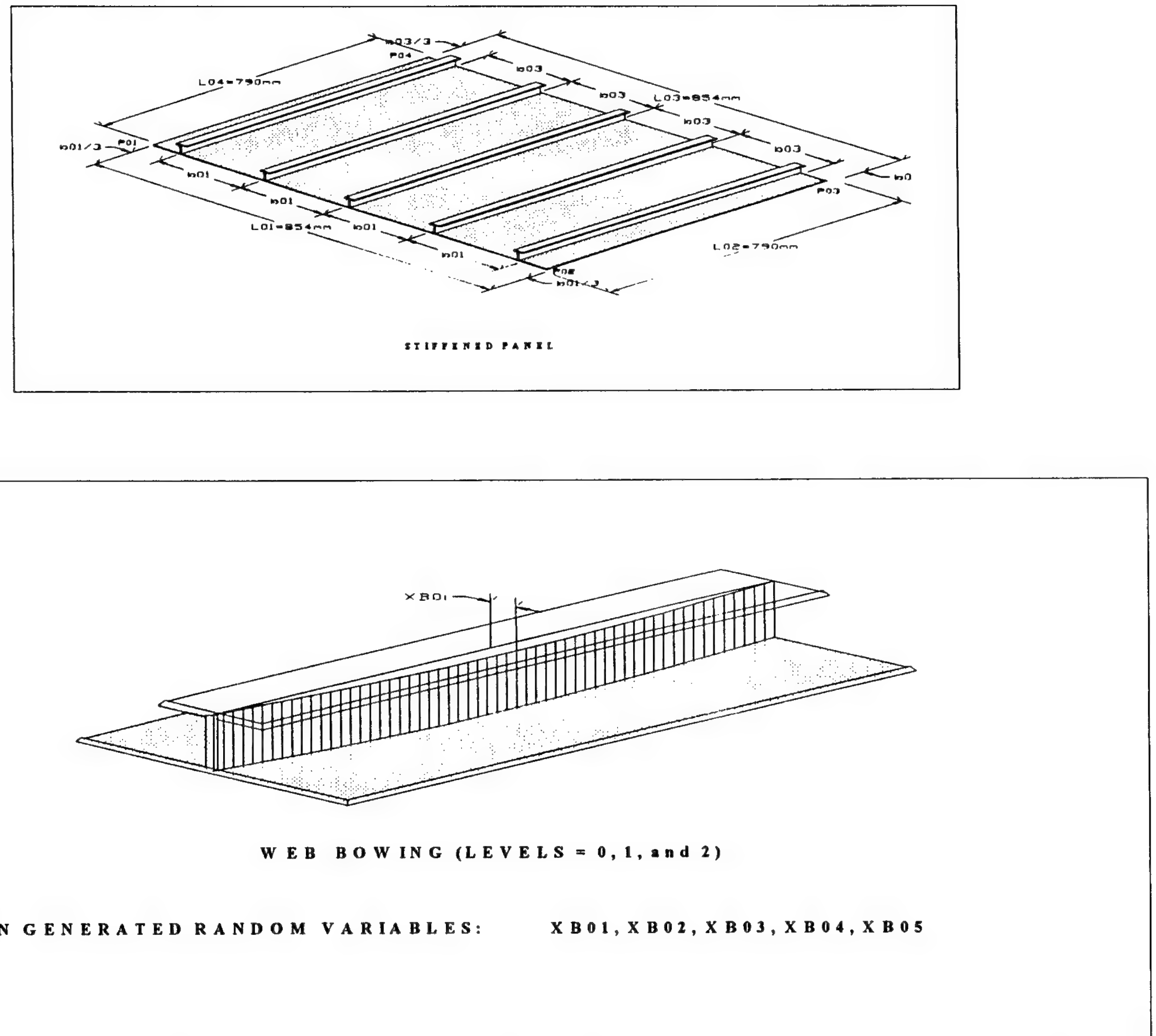



Figure 7. Web Bowing

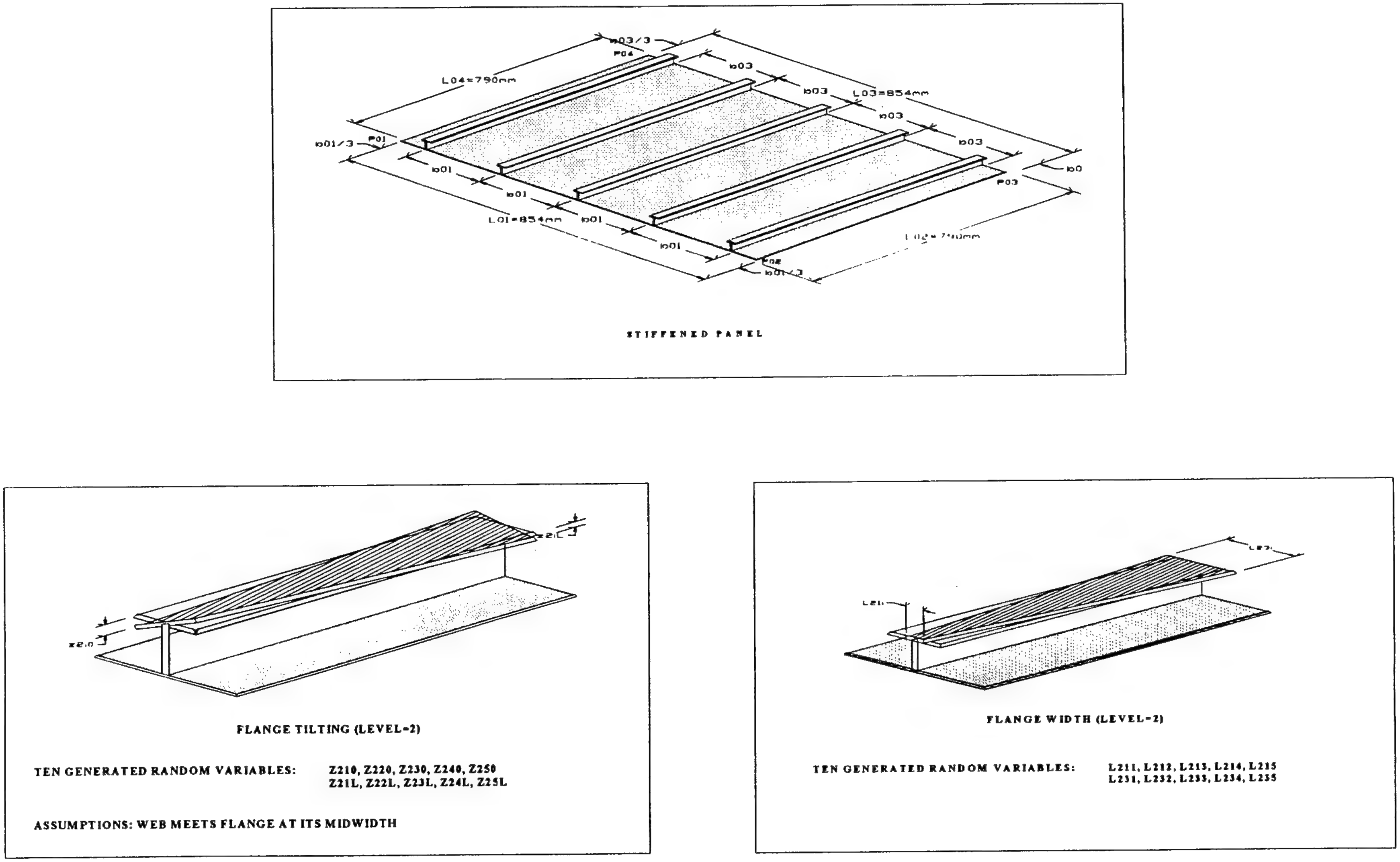



Figure 8. Flange Width Variability and Flange Tilting

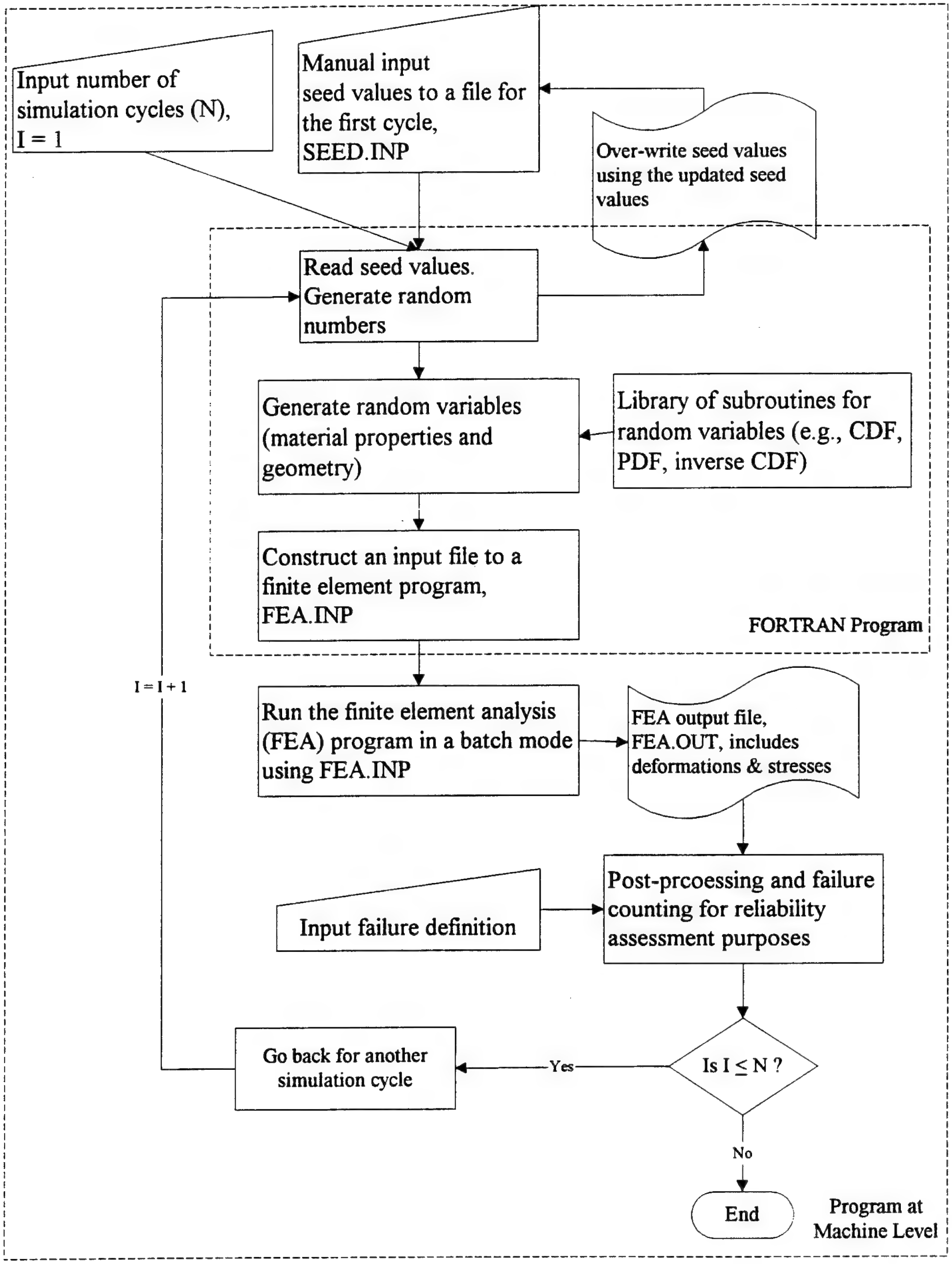



Figure 9. Methodology for *ith* Simulation Cycle

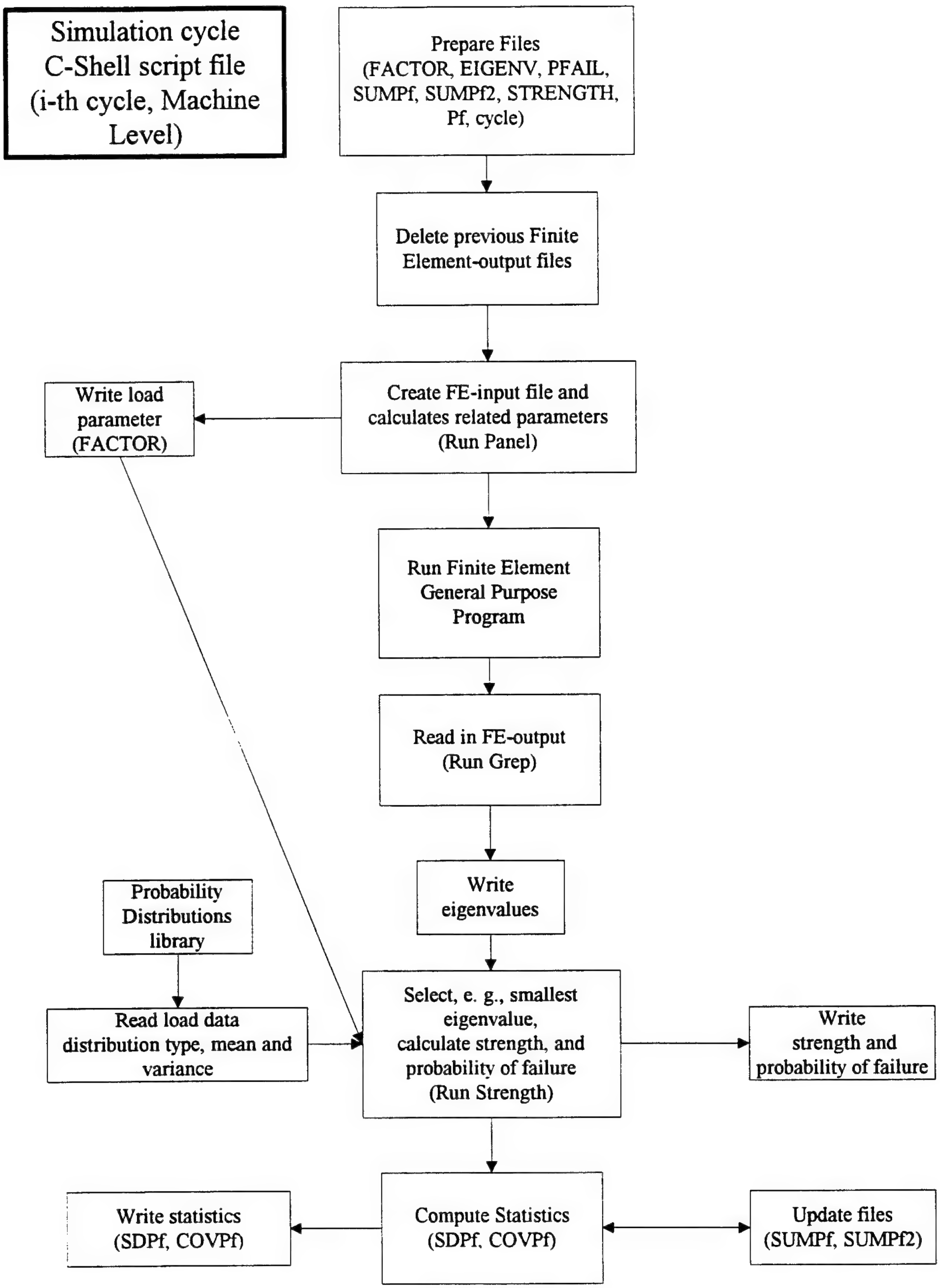



Figure 10. C-Shell Script Flow Chart for *ith* Simulation Cycle-Machine Level

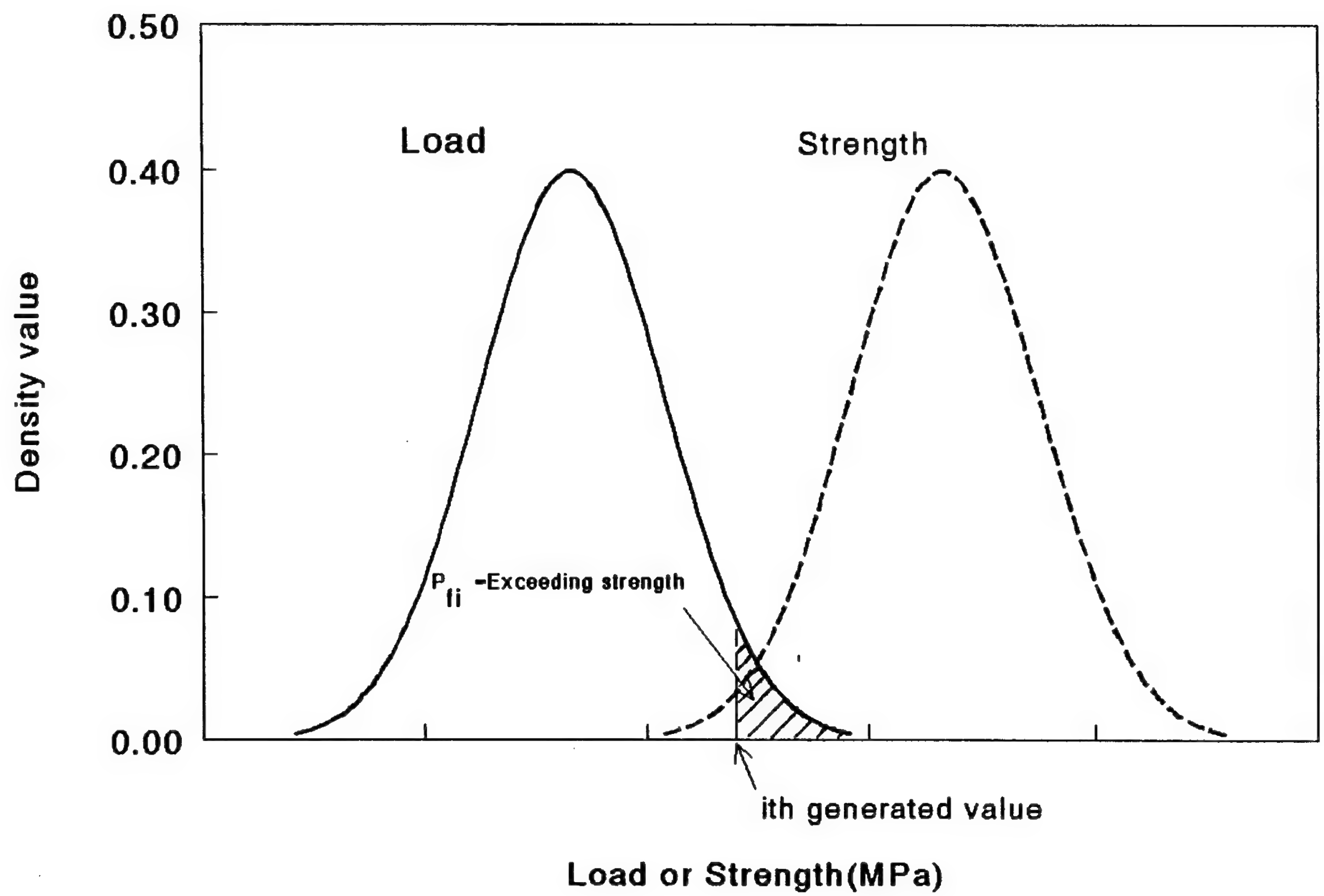
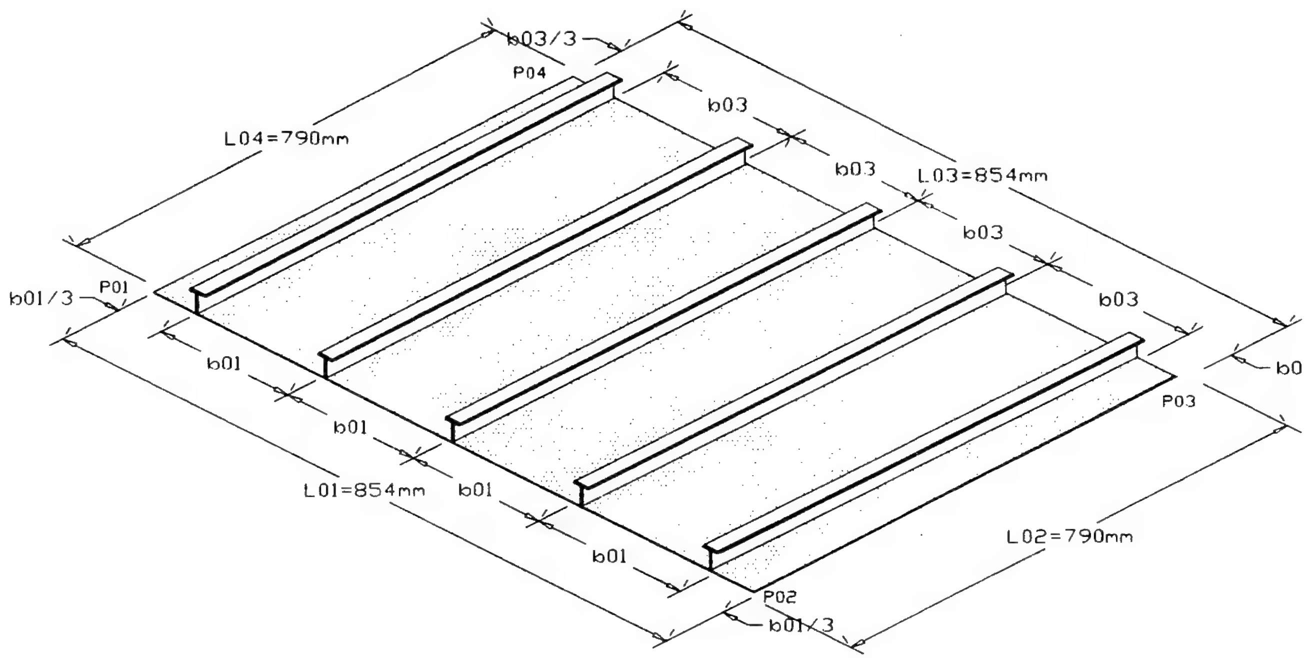



Figure 11. Conditional Expectation for Probability of Failure




**STIFFENED PANEL**

**Assumptions:** **b01 = L01 / 4.666667** **b03 = L03 / 4.666667**

Figure 12. Stiffened Panel (dimensions and assumptions)

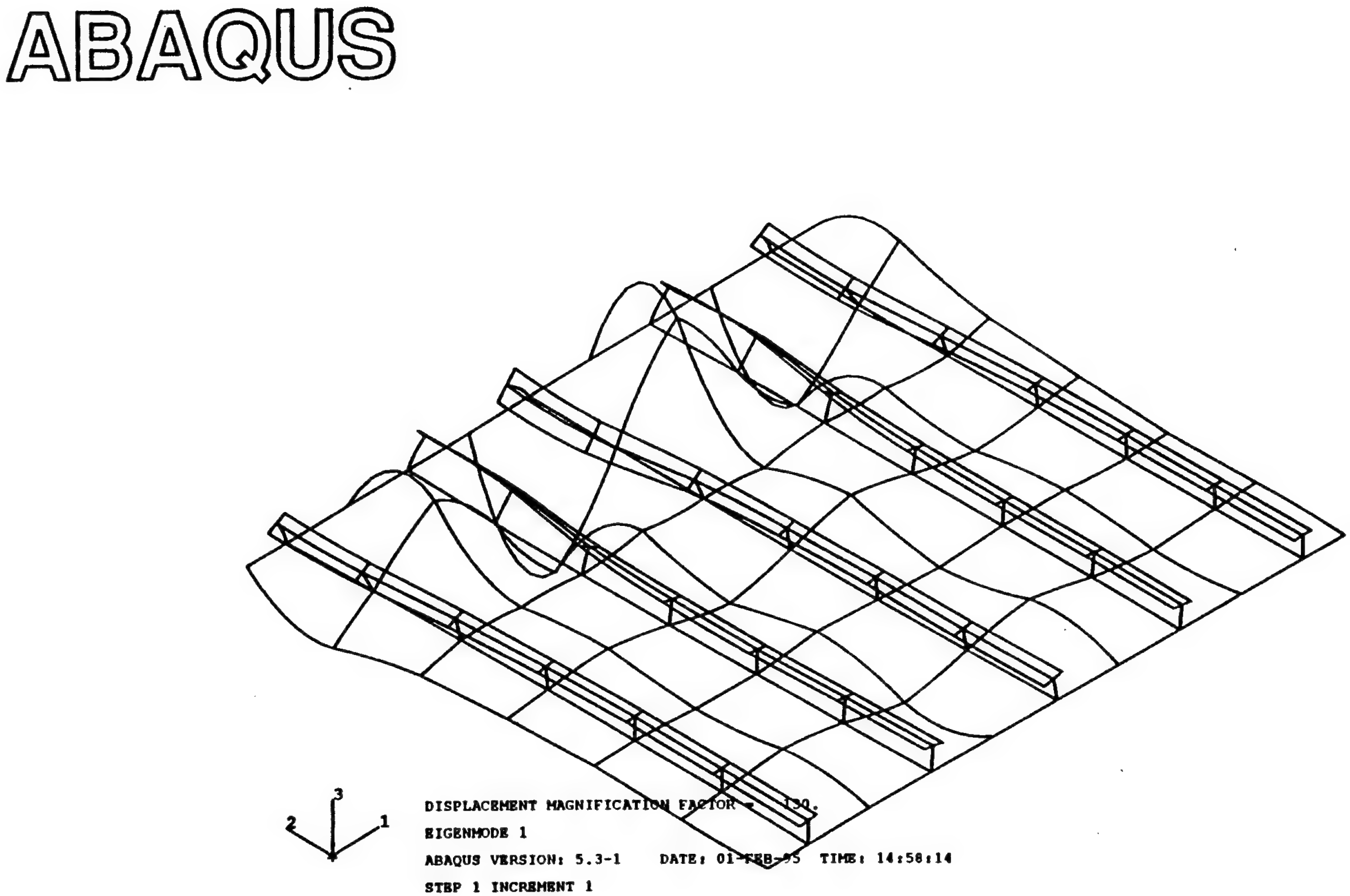
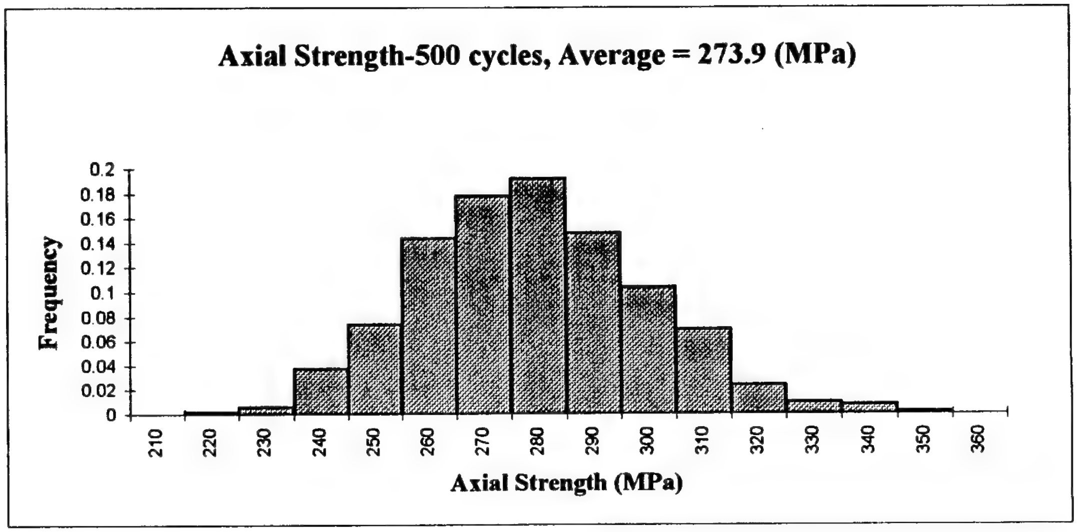



Figure 13. Buckling Shape of the Stiffened Panel

| Statistical Measures | Axial Strength |
|---|---|
| | |
| Mean | 273.9064 |
| Standard Error | 0.933275 |
| Median | 272.1663 |
| Standard Deviation | 20.86865 |
| Sample Variance | 435.5007 |
| Kurtosis | 0.031126 |
| Skewness | 0.278253 |
| Range | 121.3818 |
| Minimum | 219.5003 |
| Maximum | 340.8821 |
| Sum | 136953.2 |
| Count | 500 |
| Confidence Level(95%) | 1.829182 |




| Statistical Measures | Normalized Axial Strength |
|---|---|
| Mean | 1.00221868 |
| Standard Error | 0.00341484 |
| Median | 0.99585163 |
| Standard Deviation | 0.07635806 |
| Sample Variance | 0.00583055 |
| Kurtosis | 0.03112581 |
| Skewness | 0.27825334 |
| Range | 0.44413392 |
| Minimum | 0.80314782 |
| Maximum | 1.24728174 |
| Sum | 501.109341 |
| Count | 500 |
| Confidence Level(95%) | 0.006692946 |

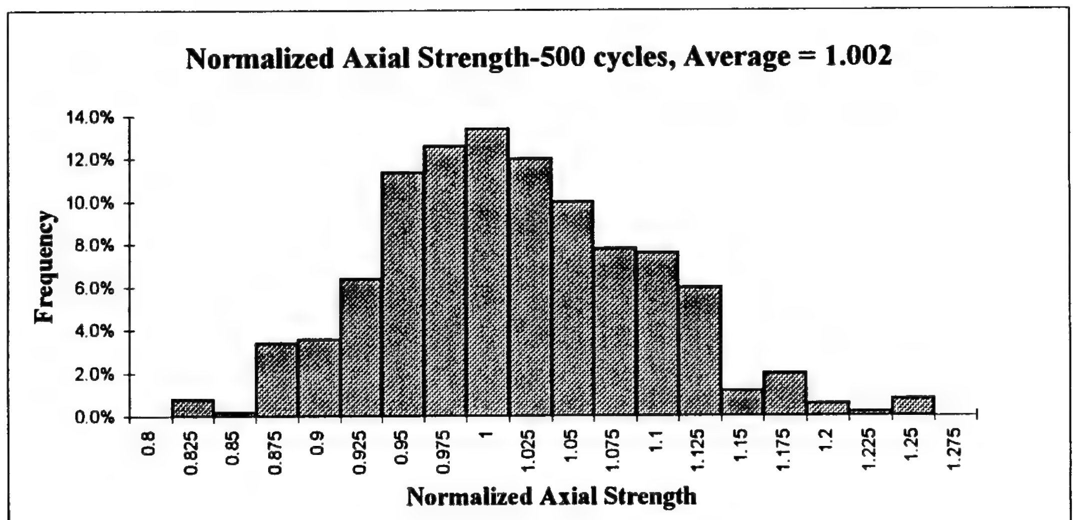
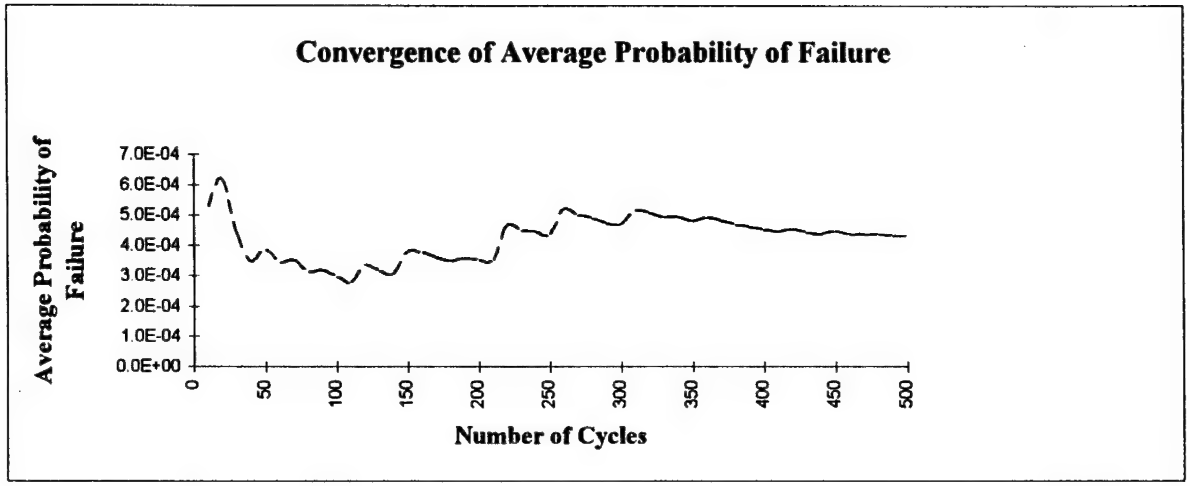



Figure 14. Axial Strength Statistics of the Stiffened Panel-500 Cycles




| Statistical Measures | Probability of Failure |
|---|---|
| | |
| Mean | 0.000431 |
| Standard Error | 8.83E-05 |
| Median | 1.83E-05 |
| Standard Deviation | 0.001973 |
| Sample Variance | 3.89E-06 |
| Kurtosis | 104.3945 |
| | 9.528373 |
| Range | 0.025693 |
| Minimum | 7.28E-11 |
| Maximum | 0.025693 |
| Sum | 0.215415 |
| Count | 500 |
| Confidence Level (95.0%) | 0.000173 |

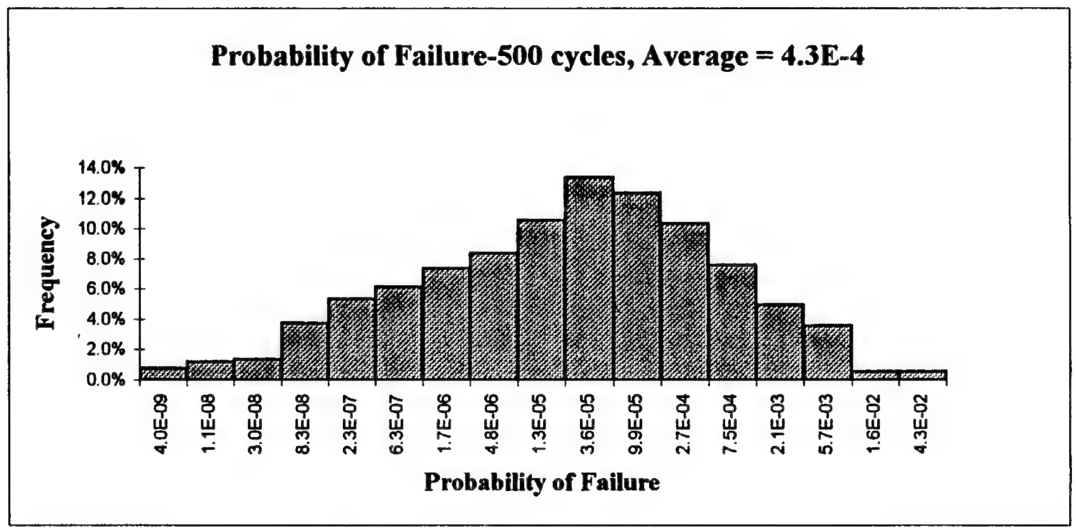
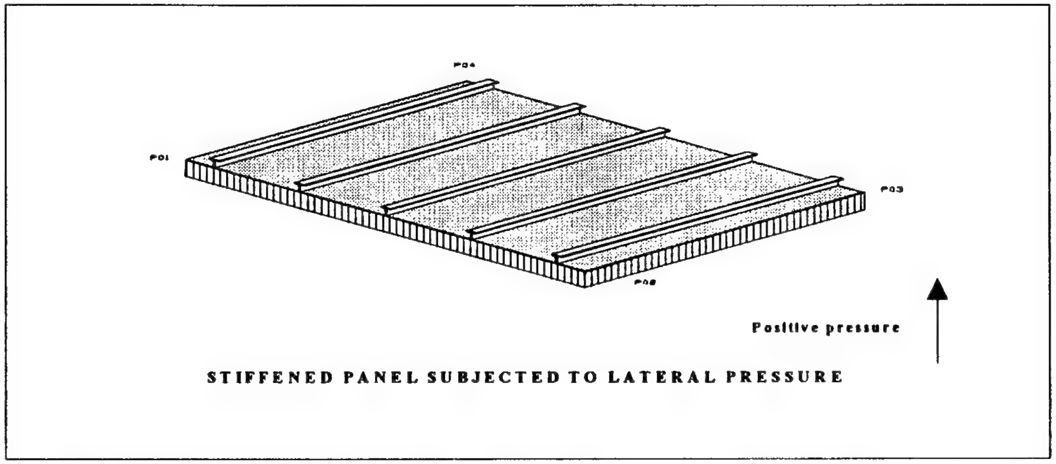



Figure 15. Probability of Failure Statistics of the Stiffened Panel-500 Cycles



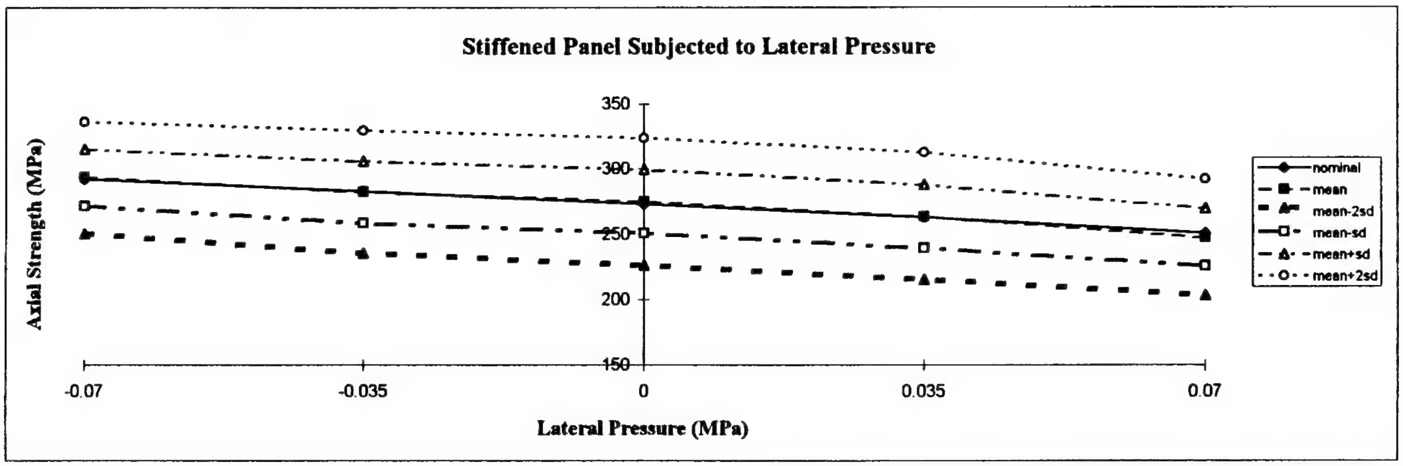
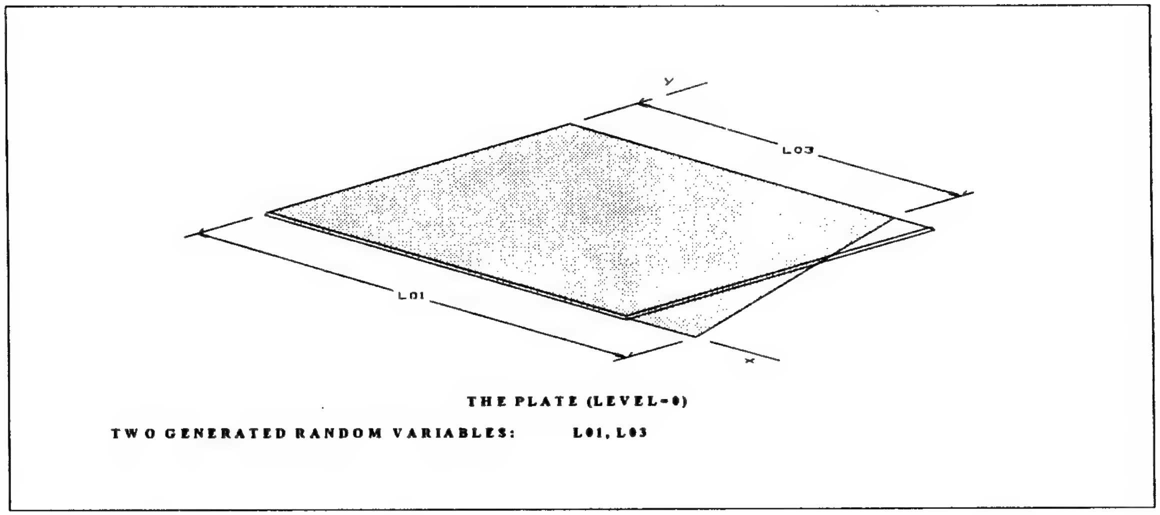
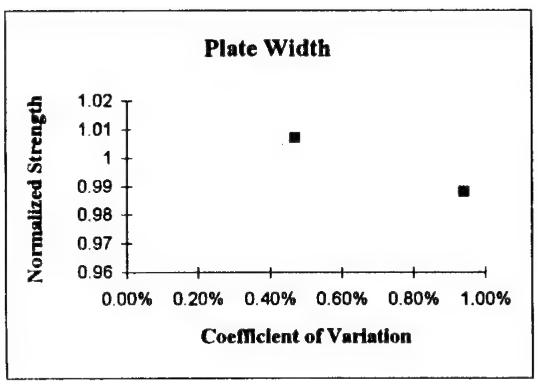
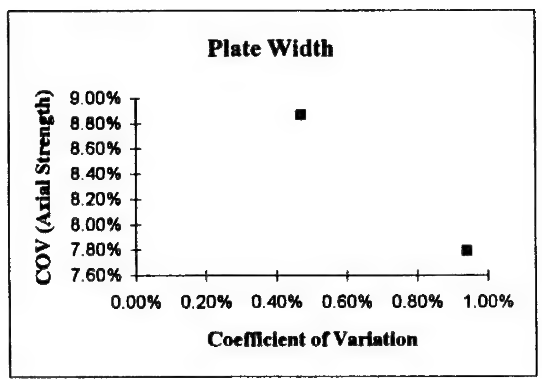




Figure 16. Stiffened Panel Subjected to Concentric Axial Compression and Lateral Pressure



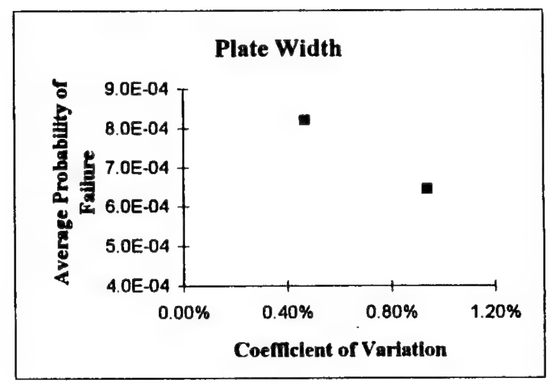




Figure 17. Strength and Probability of Failure Due to Plate Width Variability

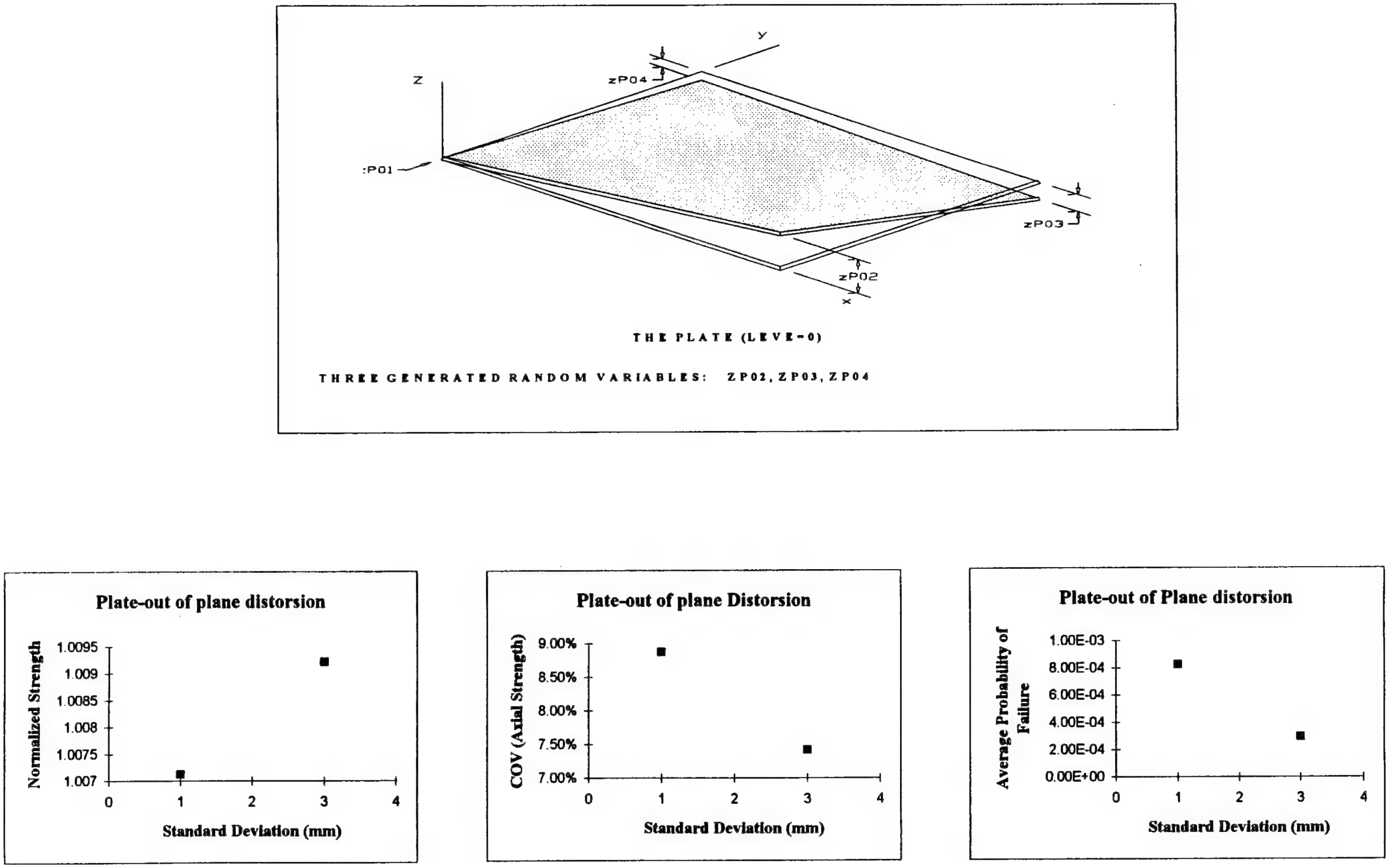



Figure 18. Strength and Probability of Failure Due to Plate Distortion Variability

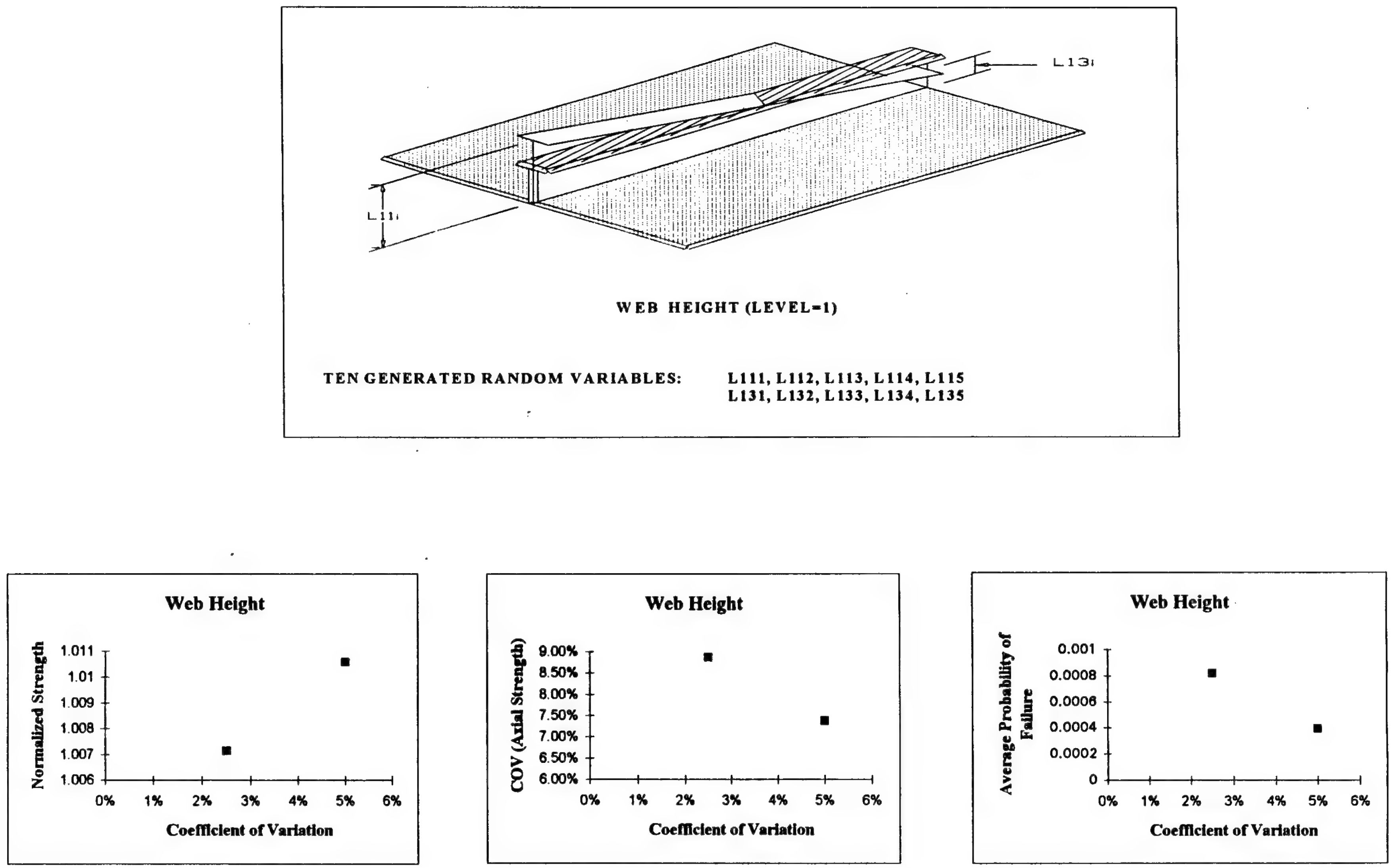
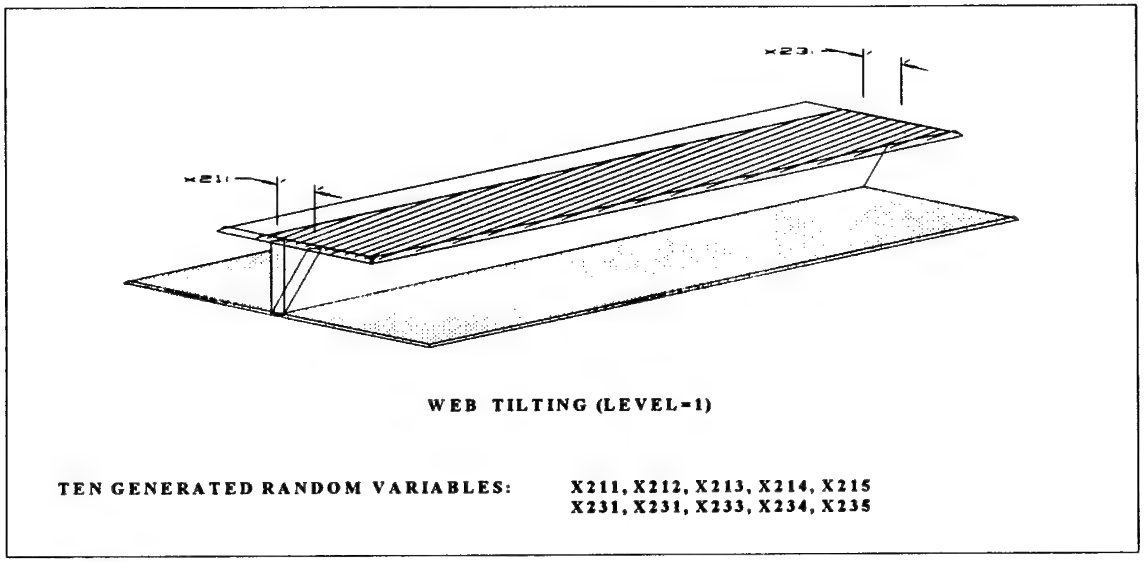
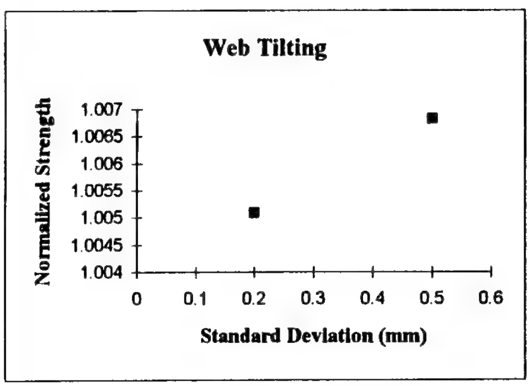
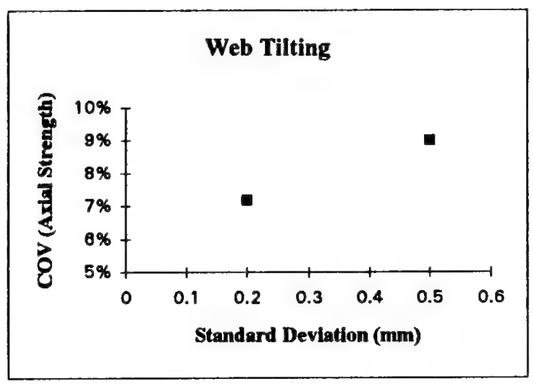



Figure 19. Strength and Probability of Failure Due to Web Height Variability



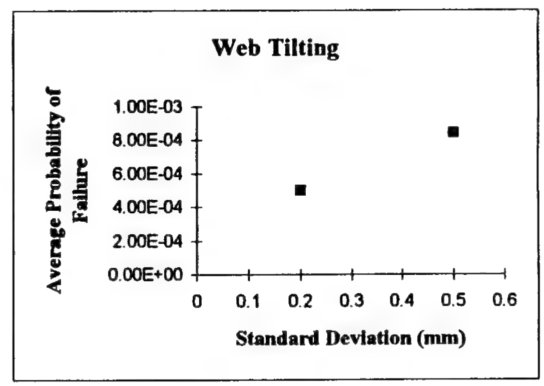
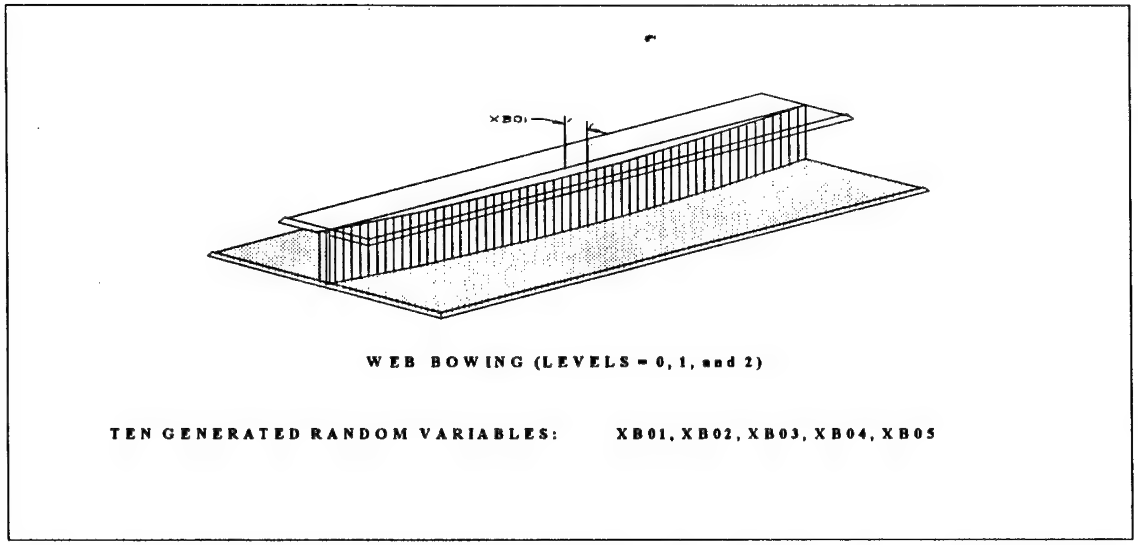
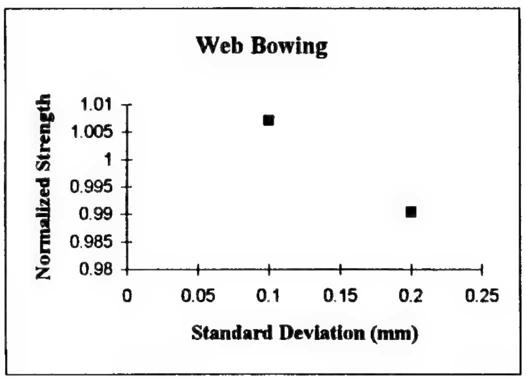
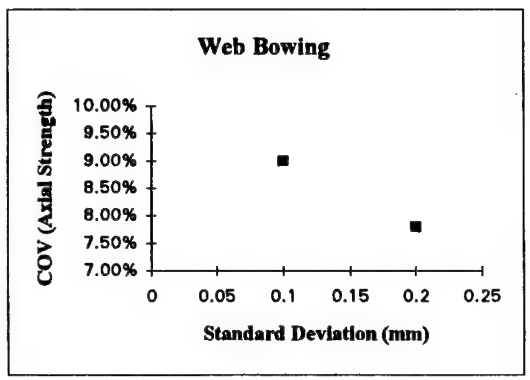




Figure 20. Strength and Probability of Failure Due to Web Tilting Variability



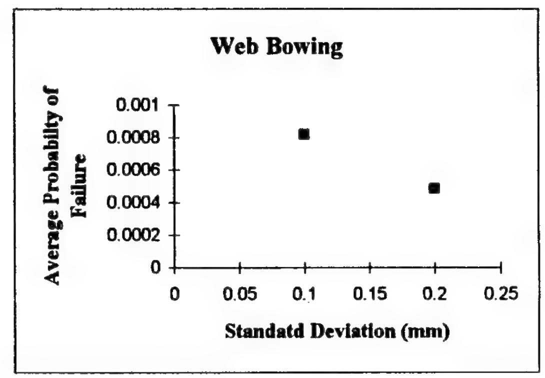




Figure 21. Strength and Probability of Failure Due to Web Bowing Variability

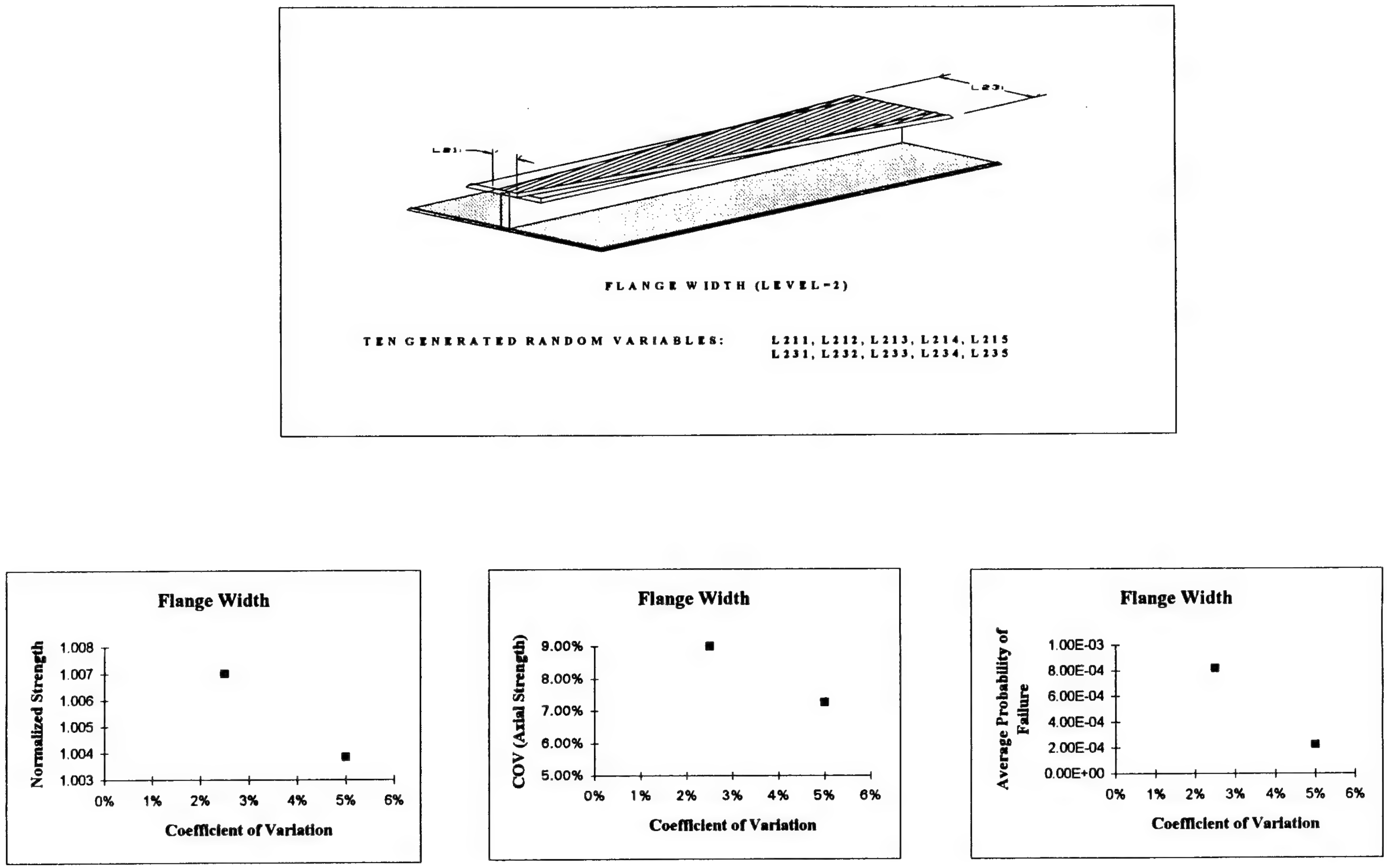
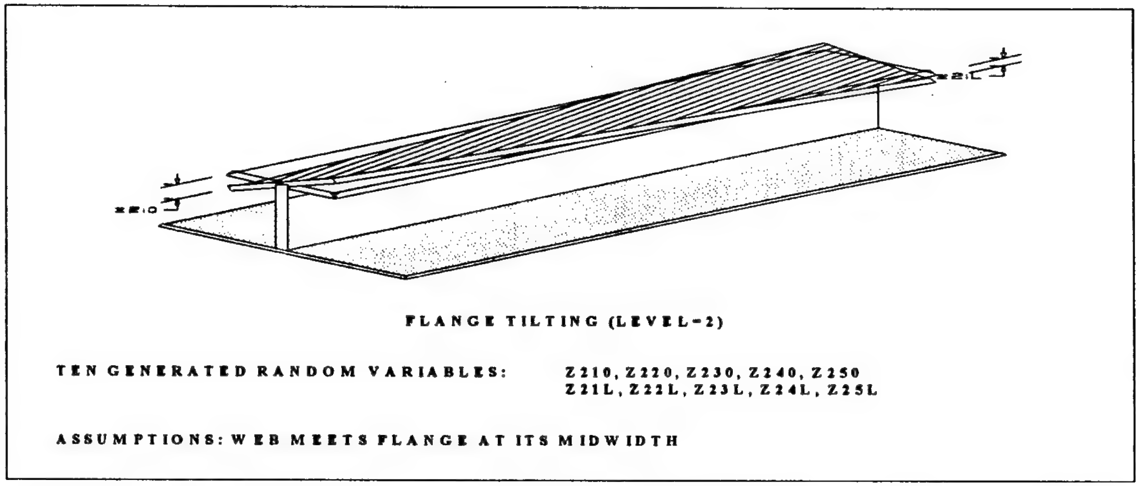
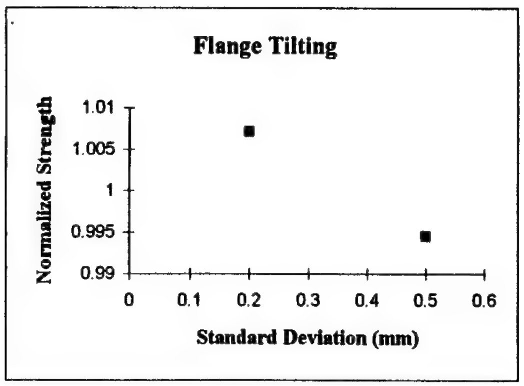
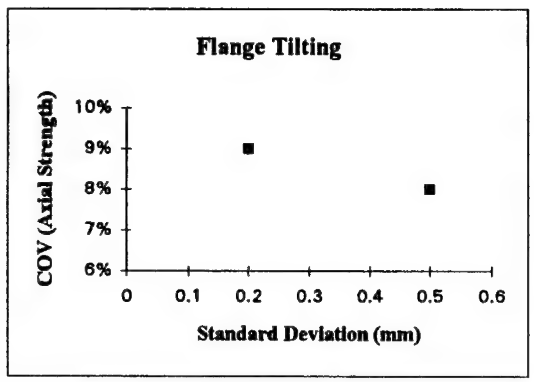



Figure 22. Strength and Probability of Failure Due to Flange Width Variability



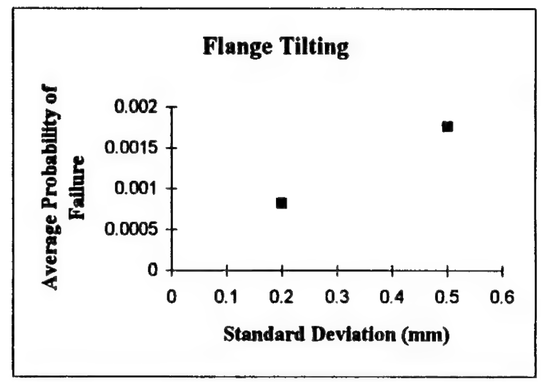




Figure 23. Strength and Probability of Failure Due to Flange Tilting Variability

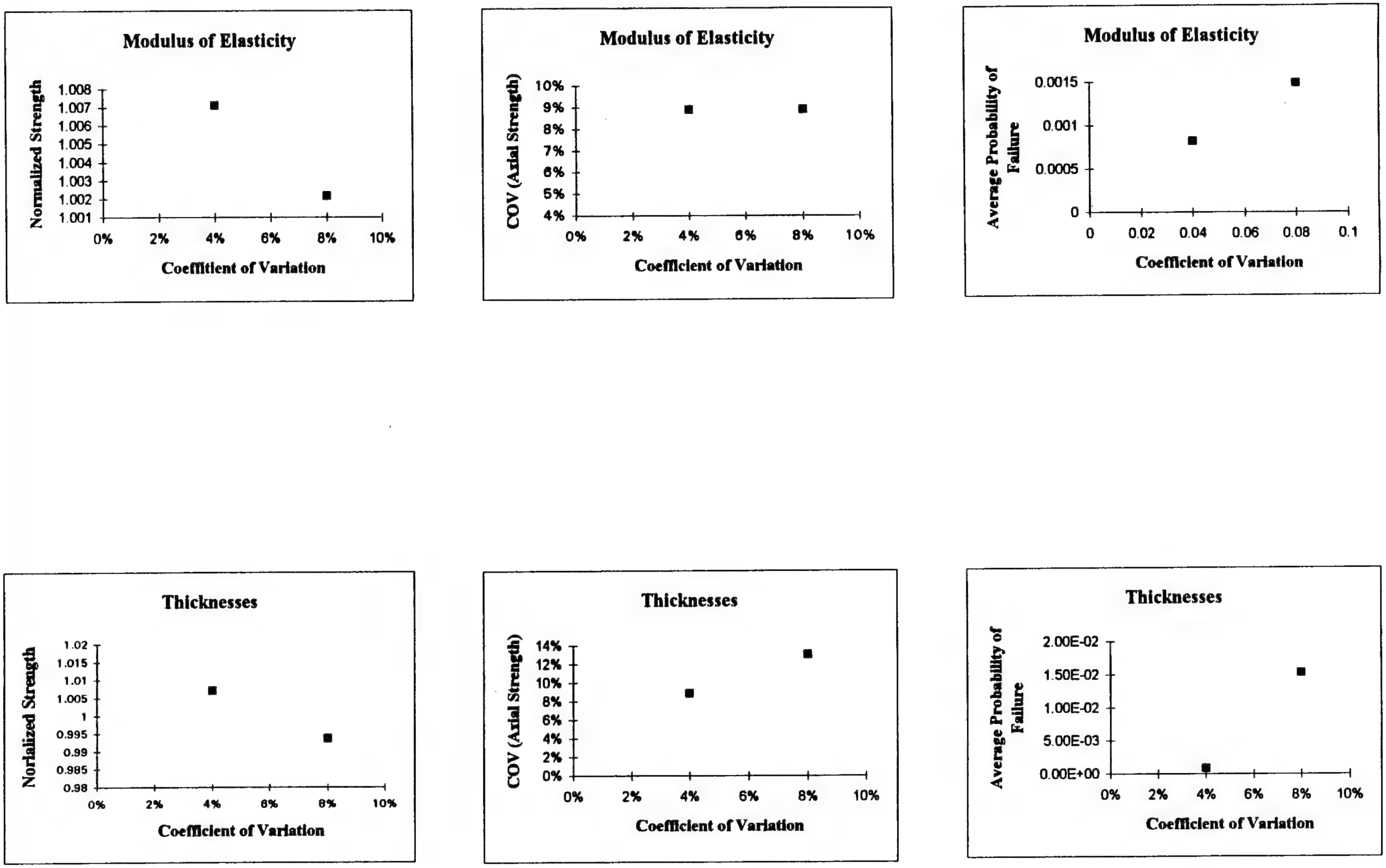



Figure 24. Strength and Probability of Failure Due to Modulus of Elasticity and Thicknesses Variability

# APPENDIX A. COMPUTER PROGRAMS

## A.1. FE-INPUT

```
PROGRAM INPUTFILE
       REAL*8 x,xx,mean,sigma,cdf
       REAL*8 H(51)
       REAL*8 L01,L03,TH0,TH1,TH2
       REAL*8 L111,L112,L113,L114,L115,L131,L132,L133,L134,L135
       REAL*8 L211,L212,L213,L214,L215,L231,L232,L233,L234,L235
       REAL*8 Z210,Z220,Z230,Z240,Z250,Z21L,Z22L,Z23L,Z24L,Z25L
       REAL*8 X211,X212,X213,X214,X215,X231,X232,X233,X234,X235
       REAL*8 ZP02,ZP03,ZP04
       REAL*8 E0,E1,E2,NU0,NU1,NU2,FY0,FY1,FY2
       REAL*8 XB01,XB02,XB03,XB04,XB05,PRESS
C...............................................................
       COMMON/LEVEL0/ L01,L03,TH0,TH1,TH2,ZP02,ZP03,ZP04,XB01,XB02,
      *               XB03,XB04,XB05,PRESS
       COMMON/PHYS/ E0,E1,E2,NU0,NU1,NU2,FY0,FY1,FY2
       COMMON/LEVEL1/ L111,L112,L113,L114,L115,L131,L132,L133,L134,L135
       COMMON/LEVEL20/ L211,L212,L213,L214,L215,Z210,Z220,Z230,Z240,Z250,
      *                X211,X212,X213,X214,X215
       COMMON/LEVEL2L/ L231,L232,L233,L234,L235,Z21L,Z22L,Z23L,Z24L,Z25L,
      *                X231,X232,X233,X234,X235
C...............................................................
       INTEGER SEED,NDIST
C...............................................................
       OPEN(UNIT=1,FILE='seed.inp',STATUS='UNKNOWN')
       REWIND(UNIT=1)
       READ(1,*) SEED
       PRINT*,'SEED = ',SEED
       CLOSE(UNIT=1)
       OPEN(UNIT=2,FILE='dist.inp',STATUS='UNKNOWN')
       REWIND(UNIT=2)
       DO 10 I=1,51
C...............................................................
       CALL RANDOM(SEED,XX)
       CDF=XX
       OPEN(UNIT=1,FILE='seed.inp',STATUS='UNKNOWN')
       REWIND(UNIT=1)
       WRITE(1,*) SEED
C
C.............DISTRIBUTIONS INPUT FILE............
C
          READ(2,*) NDIST,MEAN,SIGMA,C
C
C............. DISTRIBUTIONS................
C
C...1- NORMAL DISTRIBUTION.
C
          IF (NDIST.EQ.1) THEN
          CALL norinv(x, mean, sigma, cdf)
          ENDIF
C
C.........END DISTRIBUTIONS................
C
          H(I)=X
          L01=H(1)
          L03=H(2)
          TH0=H(3)
          TH1=H(4)
          TH2=H(5)
          ZP02=H(6)
```

```
      ZP03=H(7)
      ZP04=H(8)
      E0=H(9)
      NU0A=H(10)
      L111=H(11)
      L112=H(12)
      L113=H(13)
      L114=H(14)
      L115=H(15)
      L211=H(16)
      L212=H(17)
      L213=H(18)
      L214=H(19)
      L215=H(20)
      Z210=H(21)
      Z220=H(22)
      Z230=H(23)
      Z240=H(24)
      Z250=H(25)
      X211=H(26)
      X212=H(27)
      X213=H(28)
      X214=H(29)
      X215=H(30)
      L231=H(31)
      L232=H(32)
      L233=H(33)
      L234=H(34)
      L235=H(35)
      L131=H(36)
      L132=H(37)
      L133=H(38)
      L134=H(39)
      L135=H(40)
      Z21L=H(41)
      Z22L=H(42)
      Z23L=H(43)
      Z24L=H(44)
      Z25L=H(45)
      XB01=H(46)
      XB02=H(47)
      XB03=H(48)
      XB04=H(49)
      XB05=H(50)
      PRESS=H(51)
      CLOSE(UNIT=1)
   10 CONTINUE
      CALL INPUT
      STOP
      END
C
C.....RANDOM NUMBER GENERATOR......
C
      SUBROUTINE RANDOM(SEED,RANDX)
      INTEGER SEED
      REAL*8 RANDX
      SEED=2045*SEED+1
      SEED=SEED-(SEED/1048576)*1048576
      RANDX=REAL(SEED+1)/1048577.0
      RETURN
      END
C
C.....THIS SUBROUTINE WRITES INPUT FILE FOR ABAQUS.....
C
      SUBROUTINE INPUT
      REAL*8 A,B00,B0L,AVB0,C,L1,X121,X,L,DL,FACTOR
      REAL*8 L01,L03,TH0,TH1,TH2
```

```
      REAL*8 ZP02,ZP03,ZP04,Z01,Z03
      REAL*8 E0,E1,E2,NU0,NU1,NU2,FY0,FY1,FY2
      REAL*8 L111,L112,L113,L114,L115,L131,L132,L133,L134,L135
      REAL*8 L211,L212,L213,L214,L215,L231,L232,L233,L234,L235
      REAL*8 Z210,Z220,Z230,Z240,Z250,Z21L,Z22L,Z23L,Z24L,Z25L
      REAL*8 X211,X212,X213,X214,X215,X231,X232,X233,X234,X235
C
      REAL*8 LL00,LL1,LL2,LL10,LL20,LTOT
      REAL*8 LL111,LL112,LL113,LL114,LL115,LL131,LL132,LL133,
     *       LL134,LL135
C
      REAL*8 LL211,LL212,LL213,LL214,LL215,LL231,LL232,LL233,
     *       LL234,LL235
C
      REAL*8 A00,A01,A03,A10,A20,A1L,A2L,ATOT
      REAL*8 A111,A112,A113,A114,A115,A131,A132,A133,A134,A135
      REAL*8 A211,A212,A213,A214,A215,A231,A232,A233,A234,A235
C
      REAL*8 Q1,Q2,Q3
      REAL*8 L10,L20,L310,L320,L330,L340,L350
      REAL*8 L410,L420,L430,L440,L450
C
      REAL*8 L1L,L2L,L31L,L32L,L33L,L34L,L35L
      REAL*8 L41L,L42L,L43L,L44L,L45L
C
      REAL*8 XB01,XB02,XB03,XB04,XB05,PRESS
C...................................................................
      COMMON/LEVEL0/ L01,L03,TH0,TH1,TH2,ZP02,ZP03,ZP04,XB01,XB02,
     *               XB03,XB04,XB05,PRESS
      COMMON/PHYS/ E0,E1,E2,NU0,NU1,NU2,FY0,FY1,FY2
      COMMON/LEVEL1/ L111,L112,L113,L114,L115,L131,L132,L133,L134,L135
      COMMON/LEVEL20/ L211,L212,L213,L214,L215,Z210,Z220,Z230,Z240,Z250,
     *                X211,X212,X213,X214,X215
      COMMON/LEVEL2L/ L231,L232,L233,L234,L235,Z21L,Z22L,Z23L,Z24L,Z25L,
     *                X231,X232,X233,X234,X235
C....................................................................
      OPEN(UNIT=3,FILE='pl.inp',STATUS='UNKNOWN')
      WRITE(3,100)
 100  FORMAT('*HEADING'/
     *,'*RESTART, WRITE'/
     *,'*NODE'/
     *,'**'/
     *,'** LEVEL=0, X=0'/
     *,'**'/
     *,'101')
      C=2.*1./3.
      A=4.+(2./3.)
C
C.......CALCULATING THE SLOPE OF Y=0 AND Y=L........
C
      Z01=ZP02/L01
      Z03=(ZP03-ZP04)/L03
      B00=L01/A
      B0L=L03/A
      X102=(1./6.)*B00
      Y102=0.0
      Z102=Z01*(1./6.)*B00
      WRITE(3,110) X102,Y102,Z102
 110  FORMAT('102',',',F10.6,',',F10.6,',',F10.6)
      X103=(1./3.)*B00
      Y103=0.0
      Z103=Z01*(1./3.)*B00
      WRITE(3,120) X103,Y103,Z103
 120  FORMAT('103',',',F10.6,',',F10.6,',',F10.6)
      X107=(4./3.)*B00
      Y107=0.0
      Z107=Z01*(4./3.)*B00
```

```
      WRITE(3,121) X107,Y107,Z107
 121  FORMAT('107',',',F10.6,',',F10.6,',',F10.6)
      X111=(7./3.)*B00
      Y111=0.0
      Z111=Z01*(6./3.)*B00
      WRITE(3,122) X111,Y111,Z111
 122  FORMAT('111',',',F10.6,',',F10.6,',',F10.6)
      X115=(10./3.)*B00
      Y115=0.0
      Z115=Z01*(10./3.)*B00
      WRITE(3,123) X115,Y115,Z115
 123  FORMAT('115',',',F10.6,',',F10.6,',',F10.6)
      X119=(13./3.)*B00
      Y119=0.0
      Z119=Z01*(13./3.)*B00
      WRITE(3,130)X119,Y119,Z119
 130  FORMAT('119',',',F10.6,',',F10.6,',',F10.6)
      X120=4.5*B00
      Y120=0.0
      Z120=Z01*4.5*B00
      WRITE(3,140)X120,Y120,Z120
 140  FORMAT('120',',',F10.6,',',F10.6,',',F10.6)
      X121=L01
      Y121=0.0
      Z121=ZP02
      WRITE(3,150) X121,Y121,Z121
 150  FORMAT('121',',',F10.6,',',F10.6,',',F10.6)
C
C........COORDINATES DUE TO WEB BOWING FOR LEVEL=0 X=L/2..........
C
      WRITE(3,151)
 151  FORMAT('**'/
     *,'** LEVEL=0, X=L/2'/
     *,'**')
      AVB0=0.5*(B00+B0L)
      Z1303=ZP04+Z03*B0L/3.
      X703=(1./3.)*AVB0+XB01
      Y703=395.0
      Z703=0.5*(Z1303-Z103)+Z103
      WRITE(3,152) X703,Y703,Z703
 152  FORMAT('703',',',F10.6,',',F10.6,',',F10.6)
      Z1307=ZP04+Z03*(4./3.)*B0L
      X707=(4./3.)*AVB0+XB02
      Y707=395.0
      Z707=0.5*(Z1307-Z107)+Z107
      WRITE(3,153) X707,Y707,Z707
 153  FORMAT('707',',',F10.6,',',F10.6,',',F10.6)
      Z1311=ZP04+Z03*(7./3.)*B0L
      X711=(7./3.)*AVB0+XB03
      Y711=395.0
      Z711=0.5*(Z1311-Z111)+Z111
      WRITE(3,154) X711,Y711,Z711
 154  FORMAT('711',',',F10.6,',',F10.6,',',F10.6)
      Z1315=ZP04+Z03*(10./3.)*B0L
      X715=(10./3.)*AVB0+XB04
      Y715=395.0
      Z715=0.5*(Z1315-Z115)+Z115
      WRITE(3,155) X715,Y715,Z715
 155  FORMAT('715',',',F10.6,',',F10.6,',',F10.6)
      Z1319=ZP04+Z03*(13./3.)*B0L
      X719=(13./3.)*AVB0+XB05
      Y719=395.0
      Z719=0.5*(Z1319-Z119)+Z119
      WRITE(3,156) X719,Y719,Z719
 156  FORMAT('719',',',F10.6,',',F10.6,',',F10.6)
C
C.................................
```

```
C
      WRITE(3,160)
      L1=DSQRT(X121**2+Z121**2)
C
C.....COORDINATES OF LEVEL=0, X=L......
C
 160  FORMAT('**'/
     *,'** LEVEL=0, X=L'/
     *,'**')
      X1301=0.0
      Y1301=790.0
      Z1301=ZP04
      WRITE(3,200) X1301,Y1301,Z1301
 200  FORMAT('1301',',',F10.6,',',F10.6,',',F10.6)
      X1302=(1./6.)*B0L
      Y1302=790.0
      Z1302=ZP04+Z03*B0L/6.
      WRITE(3,210) X1302,Y1302,Z1302
 210  FORMAT('1302',',',F10.6,',',F10.6,',',F10.6)
      X1303=(1./3.)*B0L
      Y1303=790.0
      WRITE(3,220) X1303,Y1303,Z1303
 220  FORMAT('1303',',',F10.6,',',F10.6,',',F10.6)
      X1307=(4./3.)*B0L
      Y1307=790.0
      WRITE(3,221) X1307,Y1307,Z1307
 221  FORMAT('1307',',',F10.6,',',F10.6,',',F10.6)
      X1311=(7./3.)*B0L
      Y1311=790.0
      WRITE(3,222) X1311,Y1311,Z1311
 222  FORMAT('1311',',',F10.6,',',F10.6,',',F10.6)
      X1315=(10./3.)*B0L
      Y1315=790.0
      WRITE(3,223) X1315,Y1315,Z1315
 223  FORMAT('1315',',',F10.6,',',F10.6,',',F10.6)
      X1319=(13./3.)*B0L
      Y1319=790.0
      WRITE(3,230) X1319,Y1319,Z1319
 230  FORMAT('1319',',',F10.6,',',F10.6,',',F10.6)
      X1320=4.5*B0L
      Y1320=790.0
      Z1320=ZP04+Z03*(4.5*B0L)
      WRITE(3,240) X1320,Y1320,Z1320
 240  FORMAT('1320',',',F10.6,',',F10.6,',',F10.6)
      X1321=L03
      Y1321=790.0
      Z1321=ZP03
      WRITE(3,250) X1321,Y1321,Z1321
 250  FORMAT('1321',',',F10.6,',',F10.6,',',F10.6)
C
C......COORDINATE OF LEVEL=1, X=0......................
C
      WRITE(3,260)
 260  FORMAT('**'/
     *,'** LEVEL=1, X=0'/
     *,'**')
      X231=X211
      X232=X212
      X233=X213
      X234=X214
      X235=X215
      X2101=(1./3.)*B00+X211/2.
      Y2101=0.0
      Z2101=L111/2.+Z01*(1./3.)*B00
      WRITE(3,270) X2101,Y2101,Z2101
 270  FORMAT('2101',',',F10.6,',',F10.6,',',F10.6)
      X2102=(4./3.)*B00+X212/2.
```

```
      Y2102=0.0
      Z2102=L112/2.+Z01*(4./3.)*B00
      WRITE(3,271) X2102,Y2102,Z2102
 271  FORMAT('2102',',',F10.6,',',F10.6,',',F10.6)
      X2103=(7./3.)*B00+X213/2.
      Y2103=0.0
      Z2103=L113/2.+Z01*(7./3.)*B00
      WRITE(3,272) X2103,Y2103,Z2103
 272  FORMAT('2103',',',F10.6,',',F10.6,',',F10.6)
      X2104=(10./3.)*B00+X214/2.
      Y2104=0.0
      Z2104=L114/2.+Z01*(10./3.)*B00
      WRITE(3,273) X2104,Y2104,Z2104
 273  FORMAT('2104',',',F10.6,',',F10.6,',',F10.6)
      X2105=(13./3.)*B00+X215/2.
      Y2105=0.0
      Z2105=L115/2.+Z01*(13./3.)*B00
      WRITE(3,300) X2105,Y2105,Z2105
 300  FORMAT('2105',',',F10.6,',',F10.6,',',F10.6/
C
C.....COORDINATES OF LEVEL=1, Y=L/2 DUE TO BOWING....
C
     *,'**'/
     *,'** LEVEL=1, Y=L/2'/
     *,'**')
      AVLW1=0.5*(L111+L131)
      AVLW2=0.5*(L112+L132)
      AVLW3=0.5*(L113+L133)
      AVLW4=0.5*(L114+L134)
      AVLW5=0.5*(L115+L135)
C
      AVLF1=0.5*(L211+L231)
      AVLF2=0.5*(L212+L232)
      AVLF3=0.5*(L213+L233)
      AVLF4=0.5*(L214+L234)
      AVLF5=0.5*(L215+L235)
C
      X2701=(1./3.)*AVB0+X211/2.+XB01
      Y2701=395.0
      Z2701=AVLW1/2.+Z703
      WRITE(3,301) X2701,Y2701,Z2701
 301  FORMAT('2701',',',F10.6,',',F10.6,',',F10.6)
C
      X2702=(4./3.)*AVB0+X212/2.+XB02
      Y2702=395.0
      Z2702=AVLW2/2.+Z707
      WRITE(3,302) X2702,Y2702,Z2702
 302  FORMAT('2702',',',F10.6,',',F10.6,',',F10.6)
C
      X2703=(7./3.)*AVB0+X213/2.+XB03
      Y2703=395.0
      Z2703=AVLW3/2.+Z711
      WRITE(3,303) X2703,Y2703,Z2703
 303  FORMAT('2703',',',F10.6,',',F10.6,',',F10.6)
C
      X2704=(10./3.)*AVB0+X214/2.+XB04
      Y2704=395.0
      Z2704=AVLW4/2.+Z715
      WRITE(3,304) X2704,Y2704,Z2704
 304  FORMAT('2704',',',F10.6,',',F10.6,',',F10.6)
C
      X2705=(13./3.)*AVB0+X215/2.+XB05
      Y2705=395.0
      Z2705=AVLW5/2.+Z719
      WRITE(3,305) X2705,Y2705,Z2705
 305  FORMAT('2705',',',F10.6,',',F10.6,',',F10.6)
C
```

```
C.....COORDINATES OF LEVE=1, X=L..................
C
      WRITE(3,311)
 311  FORMAT('**'/
     *,'** LEVEL=1, X=L'/
     *,'**')
      X3301=(1./3.)*B0L+X231/2.
      Y3301=790.0
      Z3301=L131/2.+Z03*(1./3.)*B0L
      WRITE(3,312) X3301,Y3301,Z3301
 312  FORMAT('3301',',',F10.6,',',F10.6,',',F10.6)
      X3302=(4./3.)*B0L+X232/2.
      Y3302=790.0
      Z3302=L132/2.+Z03*(4./3.)*B0L
      WRITE(3,313) X3302,Y3302,Z3302
 313  FORMAT('3302',',',F10.6,',',F10.6,',',F10.6)
      X3303=(7./3.)*B0L+X233/2.
      Y3303=790.0
      Z3303=L133/2.+Z03*(7./3.)*B0L
      WRITE(3,314) X3303,Y3303,Z3303
 314  FORMAT('3303',',',F10.6,',',F10.6,',',F10.6)
      X3304=(10./3.)*B0L+X234/2.
      Y3304=790.0
      Z3304=L134/2.+Z03*(10./3.)*B0L
      WRITE(3,315) X3304,Y3304,Z3304
 315  FORMAT('3304',',',F10.6,',',F10.6,',',F10.6)
      X3305=(13./3.)*B0L+X235/2.
      Y3305=790.0
      Z3305=L135/2.+Z03*(13./3.)*B0L
      WRITE(3,316) X3305,Y3305,Z3305
 316  FORMAT('3305',',',F10.6,',',F10.6,',',F10.6)
C
C.....COORDINATES OF LEVEL=2, Y=0, FLANGE=1...............
C
      WRITE(3,330)
 330  FORMAT('**'/
     *,'** LEVEL=2, Y=0, FLANGE=1'/
     *,'**')
      Z211=2.*Z210/L211
      X4101=B00/3.-L211/2.+X211
      Y4101=0.0
      Z4101=Z103+L111-Z211*L211/2.
      WRITE(3,340) X4101,Y4101,Z4101
 340  FORMAT('4101',',',F10.6,',',F10.6,',',F10.6)
      X4102=B00/3.-L211/4.+X211
      Y4102=0.0
      Z4102=Z103+L111-Z211*L211/4.
      WRITE(3,350) X4102,Y4102,Z4102
 350  FORMAT('4102',',',F10.6,',',F10.6,',',F10.6)
      X4103=B00/3.+X211
      Y4103=0.0
      Z4103=Z103+L111
      WRITE(3,360) X4103,Y4103,Z4103
 360  FORMAT('4103',',',F10.6,',',F10.6,',',F10.6)
      X4104=B00/3.+L211/4.+X211
      Y4104=0.0
      Z4104=Z103+L111+Z211*L211/4.
      WRITE(3,370) X4104,Y4104,Z4104
 370  FORMAT('4104',',',F10.6,',',F10.6,',',F10.6)
      X4105=B00/3.+L211/2.+X211
      Y4105=0.0
      Z4105=Z103+L111+Z211*L211/2.
      WRITE(3,380) X4105,Y4105,Z4105
 380  FORMAT('4105',',',F10.6,',',F10.6,',',F10.6)
      X=X4105-X4101
      L=DSQRT(X**2+Z210**2)
      DL=L-X
```

```
C
C.....COORDINATES OF LEVEL=2, Y=0, FLANGE=2...............
C
      WRITE(3,390)
 390  FORMAT('**'/
     *,'** LEVEL=2, Y=0, FLANGE=2'/
     *,'**')
      Z212=2.*Z220/L212
      X4106=(4./3.)*B00-L212/2.+X212
      Y4106=0.0
      Z4106=Z107+L112-Z212*L212/2.
      WRITE(3,400) X4106,Y4106,Z4106
 400  FORMAT('4106',',',F10.6,',',F10.6,',',F10.6)
      X4107=(4./3.)*B00-L212/4.+X212
      Y4107=0.0
      Z4107=Z107+L112-Z212*L212/4.
      WRITE(3,410) X4107,Y4107,Z4107
 410  FORMAT('4107',',',F10.6,',',F10.6,',',F10.6)
      X4108=(4./3.)*B00+X212
      Y4108=0.0
      Z4108=Z107+L112
      WRITE(3,420) X4108,Y4108,Z4108
 420  FORMAT('4108',',',F10.6,',',F10.6,',',F10.6)
      X4109=(4./3.)*B00+L212/4.+X212
      Y4109=0.0
      Z4109=Z107+L112+Z212*L212/4.
      WRITE(3,430) X4109,Y4109,Z4109
 430  FORMAT('4109',',',F10.6,',',F10.6,',',F10.6)
      X4110=(4./3.)*B00+L212/2.+X212
      Y4110=0.0
      Z4110=Z107+L112+Z212*L212/2.
      WRITE(3,440) X4110,Y4110,Z4110
 440  FORMAT('4110',',',F10.6,',',F10.6,',',F10.6)
C
C.....COORDINATES OF LEVEL=2, Y=0, FLANGE=3...............
C
      WRITE(3,450)
 450  FORMAT('**'/
     *,'** LEVEL=2, Y=0, FLANGE=3'/
     *,'**')
      Z213=2*Z230/L213
      X4111=(7./3.)*B00-L213/2.
      Y4111=0.0
      Z4111=Z111+L113-Z213*L213/2.
      WRITE(3,460) X4111,Y4111,Z4111
 460  FORMAT('4111',',',F10.6,',',F10.6,',',F10.6)
      X4112=(7./3.)*B00-L213/4.+X213
      Y4112=0.0
      Z4112=Z111+L113-Z213*L213/4.
      WRITE(3,470) X4112,Y4112,Z4112
 470  FORMAT('4112',',',F10.6,',',F10.6,',',F10.6)
      X4113=(7./3.)*B00+X213
      Y4113=0.0
      Z4113=Z111+L113
      WRITE(3,480) X4113,Y4113,Z4113
 480  FORMAT('4113',',',F10.6,',',F10.6,',',F10.6)
      X4114=(7./3.)*B00+L213/4.+X213
      Y4114=0.0
      Z4114=Z111+L113+Z213*L213/4.
      WRITE(3,490) X4114,Y4114,Z4114
 490  FORMAT('4114',',',F10.6,',',F10.6,',',F10.6)
      X4115=(7./3.)*B00+L213/2.+X213
      Y4115=0.0
      Z4115=Z111+L113+Z213*L213/2.
      WRITE(3,500) X4115,Y4115,Z4115
 500  FORMAT('4115',',',F10.6,',',F10.6,',',F10.6)
C
```

```
C.....COORDINATES OF LEVEL=2, Y=0, FLANGE=4...............
C
      WRITE(3,510)
 510  FORMAT('**'/
     *,'** LEVEL=2, Y=0, FLANGE=4'/
     *,'**')
      Z214=2.*Z240/L214
      X4116=(10./3.)*B00-L214/2.+X214
      Y4116=0.0
      Z4116=Z115+L114-Z214*L214/2.
      WRITE(3,520) X4116,Y4116,Z4116
 520  FORMAT('4116',',',F10.6,',',F10.6,',',F10.6)
      X4117=(10./3.)*B00-L214/4.+X214
      Y4117=0.0
      Z4117=Z115+L114-Z214*L214/4.
      WRITE(3,530) X4117,Y4117,Z4117
 530  FORMAT('4117',',',F10.6,',',F10.6,',',F10.6)
      X4118=(10./3.)*B00+X214
      Y4118=0.0
      Z4118=Z115+L114
      WRITE(3,540) X4118,Y4118,Z4118
 540  FORMAT('4118',',',F10.6,',',F10.6,',',F10.6)
      X4119=(10./3.)*B00+L214/4.+X214
      Y4119=0.0
      Z4119=Z115+L114+Z214*L214/4.
      WRITE(3,550) X4119,Y4119,Z4119
 550  FORMAT('4119',',',F10.6,',',F10.6,',',F10.6)
      X4120=(10./3.)*B00+L214/2.+X214
      Y4120=0.0
      Z4120=Z115+L114+Z214*L214/2.
      WRITE(3,560) X4120,Y4120,Z4120
 560  FORMAT('4120',',',F10.6,',',F10.6,',',F10.6)
C
C.....COORDINATES OF LEVEL=2, Y=0, FLANGE=5...............
C
      WRITE(3,551)
 551  FORMAT('**'/
     *,'** LEVEL=2, Y=0, FLANGE=5'/
     *,'**')
      Z215=2.*Z250/L215
      X4121=(13./3.)*B00-L215/2.+X215
      Y4121=0.0
      Z4121=Z119+L115-Z215*L215/2.
      WRITE(3,570) X4121,Y4121,Z4121
 570  FORMAT('4121',',',F10.6,',',F10.6,',',F10.6)
      X4122=(13./3.)*B00-L215/4.+X215
      Y4122=0.0
      Z4122=Z119+L115-Z215*L215/4.
      WRITE(3,580) X4122,Y4122,Z4122
 580  FORMAT('4122',',',F10.6,',',F10.6,',',F10.6)
      X4123=(13./3.)*B00+X215
      Y4123=0.0
      Z4123=Z119+L115
      WRITE(3,590) X4123,Y4123,Z4123
 590  FORMAT('4123',',',F10.6,',',F10.6,',',F10.6)
      X4124=(13./3.)*B00+L215/4.+X215
      Y4124=0.0
      Z4124=Z119+L115+Z215*L215/4.
      WRITE(3,600) X4124,Y4124,Z4124
 600  FORMAT('4124',',',F10.6,',',F10.6,',',F10.6)
      X4125=(13./3.)*B00+L215/2.+X215
      Y4125=0.0
      Z4125=Z119+L115+Z215*L215/2.
      WRITE(3,610) X4125,Y4125,Z4125
 610  FORMAT('4125',',',F10.6,',',F10.6,',',F10.6/
C
C.....COORDINATES OF LEVEL=2, Y=L/2 FLANGE=1....
```

```
C
     *,'**'/
     *,'** LEVEL=2, Y=L/2'/
     *,'**')
      X4703=(1./3.)*AVB0+X211+XB01
      Y4703=395.0
      Z4703=Z703+AVLW1
      WRITE(3,611) X4703,Y4703,Z4703
 611  FORMAT('4703',',',F10.6,',',F10.6,',',F10.6)
      X4708=(4./3.)*AVB0+X212+XB02
      Y4708=395.0
      Z4708=Z707+AVLW2
      WRITE(3,612) X4708,Y4708,Z4708
 612  FORMAT('4708',',',F10.6,',',F10.6,',',F10.6)
      X4713=(7./3.)*AVB0+X213+XB03
      Y4713=395.0
      Z4713=Z711+AVLW3
      WRITE(3,613) X4713,Y4713,Z4713
 613  FORMAT('4713',',',F10.6,',',F10.6,',',F10.6)
      X4718=(10./3.)*AVB0+X214+XB04
      Y4718=395.0
      Z4718=Z715+AVLW4
      WRITE(3,614) X4718,Y4718,Z4718
 614  FORMAT('4718',',',F10.6,',',F10.6,',',F10.6)
      X4723=(13./3.)*AVB0+X215+XB05
      Y4723=395.0
      Z4723=Z719+AVLW5
      WRITE(3,615) X4723,Y4723,Z4723
 615  FORMAT('4723',',',F10.6,',',F10.6,',',F10.6)
C
C.....COORDINATES OF LEVEL=2, Y=L, FLANGE=1...............
C
      WRITE(3,620)
 620  FORMAT('**'/
     *,'** LEVEL=2, Y=L, FLANGE=1'/
     *,'**')
      Z231=2.*Z21L/L231
      X5301=B0L/3.-L231/2.+X231
      Y5301=790.0
      Z5301=Z1303+L131-Z231*L231/2.
      WRITE(3,630) X5301,Y5301,Z5301
 630  FORMAT('5301',',',F10.6,',',F10.6,',',F10.6)
      X5302=B0L/3.-L231/4.+X231
      Y5302=790.0
      Z5302=Z1303+L131-Z231*L231/4.
      WRITE(3,640) X5302,Y5302,Z5302
 640  FORMAT('5302',',',F10.6,',',F10.6,',',F10.6)
      X5303=B0L/3.+X231
      Y5303=790.0
      Z5303=Z1303+L131
      WRITE(3,650) X5303,Y5303,Z5303
 650  FORMAT('5303',',',F10.6,',',F10.6,',',F10.6)
      X5304=B0L/3.+L231/4.+X231
      Y5304=790.0
      Z5304=Z1303+L131+Z231*L231/4.
      WRITE(3,660) X5304,Y5304,Z5304
 660  FORMAT('5304',',',F10.6,',',F10.6,',',F10.6)
      X5305=B0L/3.+L231/2.+X231
      Y5305=790.0
      Z5305=Z1303+L131+Z231*L231/2.
      WRITE(3,670) X5305,Y5305,Z5305
 670  FORMAT('5305',',',F10.6,',',F10.6,',',F10.6)
C
C.....COORDINATES OF LEVEL=2, Y=L, FLANGE=2...............
C
      WRITE(3,680)
```

```
 680  FORMAT('**'/
     *,'** LEVEL=2, Y=L, FLANGE=2'/
     *,'**')
      Z232=2.*Z22L/L232
      X5306=(4./3.)*B0L-L232/2.+X232
      Y5306=790.0
      Z5306=Z1307+L132-Z232*L232/2.
      WRITE(3,690) X5306,Y5306,Z5306
 690  FORMAT('5306',',',F10.6,',',F10.6,',',F10.6)
      X5307=(4./3.)*B0L-L232/4.+X232
      Y5307=790.0
      Z5307=Z1307+L132-Z232*L232/4.
      WRITE(3,700) X5307,Y5307,Z5307
 700  FORMAT('5307',',',F10.6,',',F10.6,',',F10.6)
      X5308=(4./3.)*B0L+X232
      Y5308=790.0
      Z5308=Z1307+L132
      WRITE(3,710) X5308,Y5308,Z5308
 710  FORMAT('5308',',',F10.6,',',F10.6,',',F10.6)
      X5309=(4./3.)*B0L+L232/4.+X232
      Y5309=790.0
      Z5309=Z1307+L132+Z232*L232/4.
      WRITE(3,720) X5309,Y5309,Z5309
 720  FORMAT('5309',',',F10.6,',',F10.6,',',F10.6)
      X5310=(4./3.)*B0L+L232/2.+X232
      Y5310=790.0
      Z5310=Z1307+L132+Z232*L232/2.
      WRITE(3,730) X5310,Y5310,Z5310
 730  FORMAT('5310',',',F10.6,',',F10.6,',',F10.6)
C
C.....COORDINATES OF LEVEL=2, Y=L, FLANGE=3...............
C

      WRITE(3,740)
 740  FORMAT('**'/
     *,'** LEVEL=2, Y=L, FLANGE=3'/
     *,'**')
      Z233=2*Z23L/L233
      X5311=(7./3.)*B0L-L233/2.+X233
      Y5311=790.0
      Z5311=Z1311+L133-Z233*L233/2.
      WRITE(3,750) X5311,Y5311,Z5311
 750  FORMAT('5311',',',F10.6,',',F10.6,',',F10.6)
      X5312=(7./3.)*B0L-L233/4.+X233
      Y5312=790.0
      Z5312=Z1311+L133-Z233*L233/4.
      WRITE(3,760) X5312,Y5312,Z5312
 760  FORMAT('5312',',',F10.6,',',F10.6,',',F10.6)
      X5313=(7./3.)*B0L+X233
      Y5313=790.0
      Z5313=Z1311+L133
      WRITE(3,770) X5313,Y5313,Z5313
 770  FORMAT('5313',',',F10.6,',',F10.6,',',F10.6)
      X5314=(7./3.)*2*B0L+L233/4.+X233
      Y5314=790.0
      Z5314=Z1311+L133+Z233*L233/4.
      WRITE(3,780) X5314,Y5314,Z5314
 780  FORMAT('5314',',',F10.6,',',F10.6,',',F10.6)
      X5315=(7./3.)*B0L+L233/2.+X233
      Y5315=790.0
      Z5315=Z1311+L133+Z233*L233/2.
      WRITE(3,790) X5315,Y5315,Z5315
 790  FORMAT('5315',',',F10.6,',',F10.6,',',F10.6)
C
C.....COORDINATES OF LEVEL=2, Y=L, FLANGE=4...............
C
      WRITE(3,800)
```

```
 800  FORMAT('**'/
     *,'** LEVEL=2, Y=L, FLANGE=4'/
     *,'**')
      Z234=2.*Z24L/L234
      X5316=(10./3.)*B0L-L234/2.+X234
      Y5316=790.0
      Z5316=Z1315+L134-Z234*L234/2.
      WRITE(3,810) X5316,Y5316,Z5316
 810  FORMAT('5316',',',F10.6,',',F10.6,',',F10.6)
      X5317=(10./3.)*B0L-L234/4.+X234
      Y5317=790.0
      Z5317=Z1315+L134-Z234*L234/4.
      WRITE(3,820) X5317,Y5317,Z5317
 820  FORMAT('5317',',',F10.6,',',F10.6,',',F10.6)
      X5318=(10./3.)*B0L+X234
      Y5318=790.0
      Z5318=Z1315+L134
      WRITE(3,830) X5318,Y5318,Z5318
 830  FORMAT('5318',',',F10.6,',',F10.6,',',F10.6)
      X5319=(10./3.)*B0L+L234/4.+X234
      Y5319=790.0
      Z5319=Z1315+L134+Z234*L234/4.
      WRITE(3,840) X5319,Y5319,Z5319
 840  FORMAT('5319',',',F10.6,',',F10.6,',',F10.6)
      X5320=(10./3.)*B0L+L234/2.+X234
      Y5320=790.0
      Z5320=Z1315+L134+Z234*L234/2.
      WRITE(3,860) X5320,Y5320,Z5320
 860  FORMAT('5320',',',F10.6,',',F10.6,',',F10.6)
C
C.....COORDINATES OF LEVEL=2, Y=L, FLANGE=5................
C
      WRITE(3,850)
 850  FORMAT('**'/
     *,'** LEVEL=2, Y=L, FLANGE=5'/
     *,'**')
      Z235=2.*Z25L/L235
      X5321=(13./3.)*B0L-L235/2.+X235
      Y5321=790.0
      Z5321=Z1319+L135-Z235*L235/2.
      WRITE(3,870) X5321,Y5321,Z5321
 870  FORMAT('5321',',',F10.6,',',F10.6,',',F10.6)
      X5322=(13./3.)*B0L-L235/4.+X235
      Y5322=790.0
      Z5322=Z1319+L135-Z235*L235/4.
      WRITE(3,890) X5322,Y5322,Z5322
 890  FORMAT('5322',',',F10.6,',',F10.6,',',F10.6)
      X5323=(13./3.)*B0L+X235
      Y5323=790.0
      Z5323=Z1319+L135
      WRITE(3,900) X5323,Y5323,Z5323
 900  FORMAT('5323',',',F10.6,',',F10.6,',',F10.6)
      X5324=(13./3.)*B0L+L235/4.+X235
      Y5324=790.0
      Z5324=Z1319+L135+Z235*L235/4.
      WRITE(3,910) X5324,Y5324,Z5324
 910  FORMAT('5324',',',F10.6,',',F10.6,',',F10.6)
      X5325=(13./3.)*B0L+L235/2.+X235
      Y5325=790.0
      Z5325=Z1319+L135+Z235*L235/2.
      WRITE(3,920) X5325,Y5325,Z5325
 920  FORMAT('5325',',',F10.6,',',F10.6,',',F10.6)
C
C.....ASSINGNING LEVEL=0, Y=0, INTO SETS.........
C
      WRITE(3,1000)
 1000  FORMAT('**'/
```

```
     *,'** ASSIGNING LEVEL=0, NODES INTO SETS'/
     *,'**'/
     *,'*NGEN'/
     *,'101,103'/
     *,'*NGEN'/
     *,'103,119'/
     *,'*NGEN'/
     *,'119,121'/
     *,'*NSET, NSET=Y00EG1'/
     *,'101,102')
      WRITE(3,1010)
 1010 FORMAT('*NSET, NSET=Y00EG2'/
     *,'104,105,106'/
     *,'*NSET, NSET=Y00EG3'/
     *,'108,109,110'/
     *,'*NSET, NSET=Y00EG4'/
     *,'112,113,114'/
     *,'*NSET, NSET=Y00EG5'/
     *,'116,117,118'/
     *,'*NSET, NSET=Y00EG6'/
     *,'120,121')
C
C.....ASSINGNING LEVEL=0, Y=L, INTO SETS....
C
      WRITE(3,1011)
 1011 FORMAT('*NGEN'/
     *,'1301,1303'/
     *,'*NGEN'/
     *,'1303,1319'/
     *,'*NGEN')
      WRITE(3,1012)
 1012  FORMAT('1319,1321'/
     *,'*NSET, NSET=YL0EG1'/
     *,'1301,1302'/
     *,'*NSET, NSET=YL0EG2'/
     *,'1304,1305,1306'/
     *,'*NSET, NSET=YL0EG3'/
     *,'1308,1309,1310'/
     *,'*NSET, NSET=YL0EG4'/
     *,'1312,1313,1314'/
     *,'*NSET, NSET=YL0EG5'/
     *,'1316,1317,1318')
      WRITE(3,1015)
 1015 FORMAT('*NSET, NSET=YL0EG6'/
     *,'1320,1321'/
     *,'*NSET, NSET=YY00EG'/
     *,'Y00EG1,Y00EG2,Y00EG3,Y00EG4,Y00EG5,Y00EG6'/
     *,'*NSET, NSET=YYL0EG'/
     *,'YL0EG1,YL0EG2,YL0EG3,YL0EG4,YL0EG5,YL0EG6'/
     *,'*NFILL'/
     *,'YY00EG,YYL0EG,12,100'/
     *,'*NSET, NSET=Y00EG'/
     *,'YY00EG,103,107,111,115,119')
      WRITE(3,1020)
 1020 FORMAT('*NSET, NSET=YL0EG'/
     *,'YYL0EG,1303,1307,1311,1315,1319'/
     *,'**'/
     *,'** GENERATING LEVEL=0,  WEB LINES'/
     *,'**')
      WRITE(3,1021)
 1021 FORMAT('*NGEN,NSET=WB01,LINE=P'/
     *,'103,1303,100,703'/
     *,'*NGEN,NSET=WB02,LINE=P'/
     *,'107,1307,100,707'/
     *,'*NGEN,NSET=WB03,LINE=P'/
     *,'111,1311,100,711'/
     *,'*NGEN,NSET=WB04,LINE=P'/
```

```
     *,'115,1315,100,715'/
     *,'*NGEN,NSET=WB05,LINE=P'/
     *,'119,1319,100,719'/
     *'**'/
     *,'** LEVEL=0, X-EDGES IN SETS'/
     *,'**'/
     *,'*NGEN,NSET=X00EG')
      WRITE(3,1100)
 1100  FORMAT('101,1301,100'/
     *,'*NGEN,NSET=XL0EG'/
     *,'121,1321,100'/
     *,'**'/
C
C.....GENERATE LEVEL=1, X=0, NODES...........
C
     *,'** GENERATE LEVEL=1, X=0, NODES'/
     *,'**'/
     *,'*NSET,NSET=Y01EG'/
     *,'2101,2102,2103,2104,2105'/
     *,'*NSET,NSET=YL1EG'/
     *,'3301,3302,3303,3304,3305')
      WRITE(3,1200)
 1200  FORMAT('*NGEN,NSET=WB11,LINE=P'/
     *,'2101,3301,100,2701'/
     *,'*NGEN,NSET=WB12,LINE=P'/
     *,'2102,3302,100,2702'/
     *,'*NGEN,NSET=WB13,LINE=P'/
     *,'2103,3303,100,2703'/
     *,'*NGEN,NSET=WB14,LINE=P'/
     *,'2104,3304,100,2704'/
     *,'*NGEN,NSET=WB15,LINE=P'/
     *,'2105,3305,100,2705')
      WRITE(3,1210)
 1210 FORMAT('**'/
     *,'** GENERATE LEVEL=2, Y=0, NODES'/
     *,'**'/
     *,'*NSET, NSET=A'/
     *,'4101,4102,4104,4105'/
     *,'*NSET, NSET=B'/
     *,'4106,4107,4109,4110'/
     *,'*NSET, NSET=C'/
     *,'4111,4112,4114,4115'/
     *,'*NSET, NSET=D')
      WRITE(3,1300)
 1300 FORMAT('4116,4117,4119,4120'/
     *,'*NSET, NSET=F'/
     *,'4121,4122,4124,4125'/
     *,'*NSET, NSET=YY02EG'/
     *,'A,B,C,D,F'/
     *,'**'/
C
C.....GENERATE LEVEL=2, X=L, NODES...........
C
     *,'** GENERATE LEVEL=2, X=L, NODES'/
     *,'**'/
     *,'*NSET, NSET=G'/
     *,'5301,5302,5304,5305'/
     *,'*NSET, NSET=H')
      WRITE(3,1400)
 1400 FORMAT('5306,5307,5309,5310'/
     *,'*NSET, NSET=I'/
     *,'5311,5312,5314,5315'/
     *,'*NSET, NSET=K'/
     *,'5316,5317,5319,5320'/
     *,'*NSET, NSET=L'/
     *,'5321,5322,5324,5325'/
     *,'*NSET, NSET=YYL2EG'/
```

```
     *,'G,H,I,K,L'/
     *,'*NFILL'/
     *,'YY02EG,YYL2EG,12,100')
      WRITE(3,1410)
1410 FORMAT('*NSET, NSET=Y02EG'/
     *,'YY02EG,4103,4108,4113,4118,4123'/
     *,'*NSET, NSET=YL2EG'/
     *,'YYL2EG,5303,5308,5313,5318,5323')
      WRITE(3,1500)
1500 FORMAT('**'/
     *,'** GENERATING LEVEL=2, WEB LINES'/
     *,'**'/
     *,'*NGEN, NSET=WB21, LINE=P'/
     *,'4103,5303,100,4703'/
     *,'*NGEN, NSET=WB22, LINE=P'/
     *,'4108,5308,100,4708'/
     *,'*NGEN, NSET=WB23, LINE=P'/
     *,'4113,5313,100,4713'/
     *,'*NGEN, NSET=WB24, LINE=P'/
     *,'4118,5318,100,4718')
      WRITE(3,1510)
1510 FORMAT('*NGEN, NSET=WB25, LINE=P'/
     *,'4123,5323,100,4723'/
     *,'**'/
     *,'** ASSIGNING Y-BOUNDARY SETS'/
     *,'**'/
     *,'** 1- Y=0, Y0EDG'/
     *,'**'/
     *,'*NSET, NSET=Y0EDG'/
     *,'Y00EG,Y01EG,Y02EG'/
     *,'**'/
     *,'** 2- Y=L, YLEDG'/
     *,'**')
      WRITE(3,1600)
1600 FORMAT('*NSET, NSET=YLEDG'/
     *,'YL0EG,YL1EG,YL2EG'/
     *,'**'/
     *,'** ASSIGNING X-BOUNDARY SETS'/
     *,'**'/
     *,'** 1- X=0, X0EDG'/
     *,'**'/
     *,'*NSET, NSET=X0EDG'/
     *,'101,1301,100'/
     *,'**')
      WRITE(3,1700)
1700 FORMAT('** 2- X=L, XLEDG'/
     *,'**'/
     *,'*NSET, NSET=XLEDG'/
     *,'121,1321,100'/
     *,'**'/
     *,'** DEFINING REFERENCE ELEMENT'/
     *,'**'/
     *,'** 1- LEVEL=0'/
     *,'**'/
     *,'*ELEMENT, TYPE=S8R5, ELSET=ONE'/
     *,'1,101,103,303,301,102,203,302,201'/
     *,'*ELGEN, ELSET=ONE')
      WRITE(3,1800)
1800 FORMAT('1,10,2,1,6,200,10'/
     *,'*SHELL SECTION, MATERIAL=PLATE, ELSET=ONE')
      WRITE(3,1810) THO
1810 FORMAT(F5.3,',','3')
      WRITE(3,1820)
1820 FORMAT('**'/
     *,'** 2- LEVEL=1'/
     *,'**'/
     *,'*ELEMENT, TYPE=S8R5, ELSET=TWO'/
```

```
     *,'61,103,303,4303,4103,203,2301,4203,2101'/
     *,'*ELGEN, ELSET=TWO'/
     *,'61,1,0,0,6,200,5')
      WRITE(3,1900)
 1900 FORMAT('*ELEMENT, TYPE=S8R5, ELSET=TWO'/
     *,'62,107,307,4308,4108,207,2302,4208,2102'/
     *,'*ELGEN, ELSET=TWO'/
     *,'62,1,0,0,6,200,5'/
     *,'*ELEMENT, TYPE=S8R5, ELSET=TWO'/
     *,'63,111,311,4313,4113,211,2303,4213,2103'/
     *,'*ELGEN, ELSET=TWO'/
     *,'63,1,0,0,6,200,5'/
     *,'*ELEMENT, TYPE=S8R5, ELSET=TWO')
      WRITE(3,2000)
 2000 FORMAT('64,115,315,4318,4118,215,2304,4218,2104'/
     *,'*ELGEN, ELSET=TWO'/
     *,'64,1,0,0,6,200,5'/
     *,'*ELEMENT, TYPE=S8R5, ELSET=TWO'/
     *,'65,119,319,4323,4123,219,2305,4223,2105'/
     *,'*ELGEN, ELSET=TWO'/
     *,'65,1,0,0,6,200,5'/
     *,'*SHELL SECTION, MATERIAL=PLATE, ELSET=TWO')
      WRITE(3,2110) TH1
 2110 FORMAT(F5.3,',','3')
      WRITE(3,2120)
 2120 FORMAT('**')
      WRITE(3,2200)
 2200 FORMAT('**'/
     *,'** 3- LEVEL=2'/
     *,'**'/
     *,'*ELEMENT, TYPE=S8R5, ELSET=THREE'/
     *,'91,4101,4103,4303,4301,4102,4203,4302,4201'/
     *,'*ELGEN, ELSET=THREE'/
     *,'91,5,5,2,6,200,10'/
     *,'*ELEMENT, TYPE=S8R5, ELSET=THREE'/
     *,'92,4103,4105,4305,4303,4104,4205,4304,4203'/
     *,'*ELGEN, ELSET=THREE'/
     *,'92,5,5,2,6,200,10')
      WRITE(3,2300)
 2300 FORMAT('*SHELL SECTION, MATERIAL=PLATE, ELSET=THREE')
      WRITE(3,2310) TH2
 2310 FORMAT(F5.3,',','3')
      WRITE(3,2320)
 2320 FORMAT('**'/
     *,'*MATERIAL, NAME=PLATE'/
     *,'*ELASTIC')
      WRITE(3,2330) E0,NU0
 2330 FORMAT(F8.1,',',F4.2)
      WRITE(3,2340)
 2340 FORMAT('*BOUNDARY'/
     *,'Y00EG,1'/
     *,'Y00EG,3'/
     *,'Y00EG,5')
      WRITE(3,2400)
 2400 FORMAT('Y00EG,6'/
     *,'YL0EG,1'/
     *,'YL0EG,2'/
     *,'YL0EG,3'/
     *,'YL0EG,5'/
     *,'YL0EG,6'/
     *,'*DRAW, ELNUM'/
     *,'*DRAW, NODENUM')
      WRITE(3,3100)
 3100 FORMAT('**'/
     *,'*STEP'/
     *,'*STATIC'/
     *,'*DLOAD')
```

```
      WRITE(3,3101) PRESS
 3101 FORMAT('ONE,P,',F7.5/
     *,'*END STEP'/
     *,'*STEP'/
     *,'*BUCKLE')
C
C.......CACULATIONS OF NODAL FORCES...................
C
C......  1- LEVEL=0, X=0  ...........
C
      Q1=1
      Q2=Q1*TH1/TH0
      Q3=Q1*TH2/TH0
      L10=B00/3.
      P101=Q1*L10/6.
      P102=Q1*4*(L10/6.)
      P103L=Q1*L10/6.
      L20=B00/2.
      P103R=Q1*L20/6.
      P103B=P103L+P103R
      P104=Q1*4*(L20/6.)
      P105=2*Q1*(L20/6.)
      P106=P104
      P107B=2*Q1*(L20/6.)
      P108=P104
      P109=P105
      P110=P104
      P111B=P107B
      P112=P104
      P113=P105
      P114=P104
      P115B=P107B
      P116=P104
      P117=P105
      P118=P104
      P119B=P103B
      P120=P102
      P121=P101
C
C......  LEVEL=1, X=0  .................
C
      L310=L111
      P103T=Q2*L310/6.
      P2101=4*Q2*L310/6.
      P4103B=P103T
      L320=L112
      P107T=Q2*L320/6.
      P2102=4*Q2*L320/6.
      P4108B=P107T
      L330=L113
      P111T=Q2*L330/6.
      P2103=4*Q2*L330/6.
      P4113B=P111T
      L340=L114
      P115T=Q2*L340/6.
      P2104=4*Q2*L340/6.
      P4118B=P115T
      L350=L115
      P119T=Q2*L350/6.
      P2105=4*Q2*L350/6.
      P4123B=P119T
C
C.......  LEVEL=2, X=0  .............
C
      L410=L211/2.
      P4101=Q3*L410/6.
      P4102=4*Q3*L410/6.
```

```
      P4103T=2*Q3*L410/6.
      P4104=P4102
      P4105=P4101
      L420=L212/2.
      P4106=Q3*L420/6.
      P4107=4*Q3*L420/6.
      P4108T=2*Q3*L420/6.
      P4109=P4107
      P4110=P4106
      L430=L213/2.
      P4111=Q3*L430/6.
      P4112=4*Q3*L430/6.
      P4113T=2*Q3*L430/6.
      P4114=P4112
      P4115=P4111
      L440=L214/2.
      P4116=Q3*L440/6.
      P4117=4*Q3*L440/6.
      P4118T=2*Q3*L440/6.
      P4119=P4117
      P4120=P4116
      L450=L215/2.
      P4121=Q3*L450/6.
      P4122=4*Q3*L450/6.
      P4123T=2*Q3*L450/6.
      P4124=P4122
      P4125=P4121
C
C....... FINAL LOAD VALUES, X=0 ..........
C
      P103=P103B+P103T
      P107=P107B+P107T
      P111=P111B+P111T
      P115=P115B+P115T
      P119=P119B+P119T
      P4103=P4103B+P4103T
      P4108=P4108B+P4108T
      P4113=P4113B+P4113T
      P4118=P4118B+P4118T
      P4123=P4123B+P4123T
C
C......  1- LEVEL=0, X=L  ...........
C
      L1L=B00/3.
      P1301=-Q1*L1L/6.
      P1302=-Q1*4*(L1L/6.)
      P1303L=-Q1*L1L/6.
      L2L=B00/2.
      P1303R=-Q1*L2L/6.
      P1303B=-(P103L+P103R)
      P1304=-Q1*4*(L2L/6.)
      P1305=-2*Q1*(L2L/6.)
      P1306=-P104
      P1307B=-2*Q1*(L2L/6.)
      P1308=-P104
      P1309=-P105
      P1310=-P104
      P1311B=-P107B
      P1312=-P104
      P1313=-P105
      P1314=-P104
      P1315B=-P107B
      P1316=-P104
      P1317=-P105
      P1318=-P104
      P1319B=-P103B
      P1320=-P102
```

```
      P1321=-P101
C
C......  LEVEL=1, X=L  .................
C
      L31L=L111
      P1303T=-Q3*L31L/6.
      P3301=-4*Q2*L31L/6.
      P5303B=P1303T
      L32L=L112
      P1307T=-Q2*L32L/6.
      P3302=-4*Q2*L32L/6.
      P5308B=P1307T
      L33L=L113
      P1311T=-Q2*L33L/6.
      P3303=-4*Q2*L33L/6.
      P5313B=P1311T
      L34L=L114
      P1315T=-Q2*L34L/6.
      P3304=-4*Q2*L34L/6.
      P5318B=P1315T
      L35L=L115
      P1319T=-Q2*L35L/6.
      P3305=-4*Q2*L35L/6.
      P5323B=P1319T
C
C.......  LEVEL=2, X=L  ............
C
      L41L=L211/2.
      P5301=-Q3*L41L/6.
      P5302=-4*Q3*L41L/6.
      P5303T=-2*Q3*L41L/6.
      P5304=P5302
      P5305=P5301
      L42L=L212/2.
      P5306=-Q3*L42L/6.
      P5307=-4*Q3*L42L/6.
      P5308T=-2*Q3*L42L/6.
      P5309=P5307
      P5310=P5306
      L43L=L213/2.
      P5311=-Q3*L43L/6.
      P5312=-4*Q3*L43L/6.
      P5313T=-2*Q3*L43L/6.
      P5314=P5312
      P5315=P5311
      L44L=L214/2.
      P5316=-Q3*L44L/6.
      P5317=-4*Q3*L44L/6.
      P5318T=-2*Q2*L44L/6.
      P5319=P5317
      P5320=P5316
      L45L=L215/2.
      P5321=-Q3*L45L/6.
      P5322=-4*Q3*L45L/6.
      P5323T=-2*Q3*L45L/6.
      P5324=P5322
      P5325=P5321
C
C....... FINAL LOAD VALUES, X=L .........
C
      P1303=P1303B+P1303T
      P1307=P1307B+P1307T
      P1311=P1311B+P1311T
      P1315=P1315B+P1315T
      P1319=P1319B+P1319T
      P5303=P5303B+P5303T
      P5308=P5308B+P5308T
```

```
      P5313=P5313B+P5313T
      P5318=P5318B+P5318T
      P5323=P5323B+P5323T
C
C....... CALCULATING TOTAL APPLIED AXIAL LOAD .........
C
      LL00=L01+L03
      A01=L01*TH0
      A03=L03*TH0
      A00=A01+A03
      LL1=TH1/TH0
      LL2=TH2/TH0
C
C.....THE WEB, X=0.....
C
      LL111=L111*LL1
      LL112=L112*LL1
      LL113=L113*LL1
      LL114=L114*LL1
      LL115=L115*LL1
C..........................................................
      A111=L111*TH1
      A112=L112*TH1
      A113=L113*TH1
      A114=L114*TH1
      A115=L115*TH1
      A10=A111+A112+A113+A114+3*A115
C..........................................................
      LL211=L211*LL2
      LL212=L212*LL2
      LL213=L213*LL2
      LL214=L214*LL2
      LL215=L215*LL2
C..........................................................
      A211=L211*TH2
      A212=L212*TH2
      A213=L213*TH2
      A214=L214*TH2
      A215=L215*TH2
      A20=A211+A212+A213+A214+A215
C..........................................................
      LL131=L131*LL1
      LL132=L132*LL1
      LL133=L133*LL1
      LL134=L134*LL1
      LL135=L135*LL1
C..........................................................
      A131=L131*TH1
      A132=L132*TH1
      A133=L133*TH1
      A134=L134*TH1
      A135=L135*TH1
      A1L=A131+A132+A133+A134+A135
C..........................................................
      LL231=L231*LL2
      LL232=L232*LL2
      LL233=L233*LL2
      LL234=L234*LL2
      LL235=L235*LL2
C..........................................................
      A231=L231*TH2
      A232=L232*TH2
      A233=L233*TH2
      A234=L234*TH2
      A235=L235*TH2
      A2L=A231+A232+A233+A234+A235
C..........................................................
```

```
      LL10=LL111+LL112+LL113+LL114+LL115+LL131+LL132+LL133+
     *LL134+LL135
C.............................................................
      LL20=LL211+LL212+LL213+LL214+LL215+LL231+LL232+LL233+
     *LL234+LL235
C.............................................................
      LTOT=LL00+LL10+LL20
      ATOT=A00+2*A10+A20+A1L+A2L
C
      FACTOR=LTOT/ATOT
C
      PRINT*,'FACTOR=',FACTOR
      WRITE(3,2500)
 2500 FORMAT('1,'/
     *,'**'/
     *,'*MODAL FILE'/
     *,'**'/
     *,'** LEVEL=0, X=0, LOAD SETS'/
     *,'**'/
     *,'*NSET, NSET=LOAD001'/
     *,'101,121'/
     *,'*NSET, NSET=LOAD002'/
     *,'102,120')
      WRITE(3,2600)
 2600 FORMAT('*NSET, NSET=LOAD003'/
     *,'103,119')
      WRITE(3,2610)
 2610 FORMAT('*NSET, NSET=LOAD004'/
     *,'104,106,108,110,112,114,116,118'/
     *,'*NSET, NSET=LOAD005'/
     *,'105,109,113,117'/
     *,'*NSET, NSET=LOAD006'/
     *,'107,111,115')
      WRITE(3,2700)
 2700 FORMAT('**'/
     *,'** LEVEL=2, X=0, LOAD SETS'/
     *,'**'/
     *,'*NSET, NSET=LOAD021'/
     *,'4101,4105'/
     *,'*NSET, NSET=LOAD022'/
     *,'4102,4104'/
     *,'*NSET, NSET=LOAD023'/
     *,'4106,4110')
      WRITE(3,2710)
 2710 FORMAT('*NSET, NSET=LOAD024'/
     *,'4107,4109'/
     *,'*NSET, NSET=LOAD025'/
     *,'4111,4115'/
     *,'*NSET, NSET=LOAD026'/
     *,'4112,4114'/
     *,'*NSET, NSET=LOAD027'/
     *,'4116,4120'/
     *,'*NSET, NSET=LOAD028'/
     *,'4117,4119')
      WRITE(3,2720)
 2720 FORMAT('*NSET, NSET=LOAD029'/
     *,'4121,4125'/
     *,'*NSET, NSET=LOAD0210'/
     *,'4122,4124')
      WRITE(3,2800)
 2800 FORMAT('**'/
     *,'** LEVEL=0, X=L, LOAD SETS'/
     *,'**'/
     *,'*NSET, NSET=LOADL01'/
     *,'1301,1321'/
     *,'*NSET, NSET=LOADL02'/
     *,'1302,1320'/
```

```
     *,'*NSET, NSET=LOADL03')
      WRITE(3,2900)
2900 FORMAT('1303,1319'/
     *,'*NSET, NSET=LOADL04'/
     *,'1304,1306,1308,1310,1312,1314,1316,1318'/
     *,'*NSET, NSET=LOADL05'/
     *,'1305,1309,1313,1317'/
     *,'*NSET, NSET=LOADL06'/
     *,'1307,1311,1315'/
     *,'**'/
     *,'** LEVEL=1, X=L, LOAD SETS'/
     *,'**')
      WRITE(3,3000)
3000 FORMAT('*NSET, NSET=LOADL1'/
     *,'3301,3302,3303,3304,3305'/
     *,'**'/
     *,'** LEVEL=2, X=L, LOAD SETS'/
     *,'**'/
     *,'*NSET, NSET=LOADL21'/
     *,'5301,5305'/
     *,'*NSET, NSET=LOADL22'/
     *,'5302,5304'/
     *,'*NSET, NSET=LOADL23')
      WRITE(3,3010)
3010 FORMAT('5306,5310'/
     *,'*NSET, NSET=LOADL24'/
     *,'5307,5309'/
     *,'*NSET, NSET=LOADL25'/
     *,'5311,5315'/
     *,'*NSET, NSET=LOADL26'/
     *,'5312,5314'/
     *,'*NSET, NSET=LOADL27'/
     *,'5316,5320'/
     *,'*NSET, NSET=LOADL28')
      WRITE(3,3020)
3020 FORMAT('5317,5319'/
     *,'*NSET, NSET=LOADL29'/
     *,'5321,5325'/
     *,'*NSET, NSET=LOADL210'/
     *,'5322,5324')
      WRITE(3,3030)
3030 FORMAT('*CLOAD'/
     *,'**'/
     *,'**LEVEL=0, X=0'/
     *,'**')
      WRITE(3,3102)P101
3102 FORMAT('LOAD001,2,',F10.6)
      WRITE(3,3103)P102
3103 FORMAT('LOAD002,2,',F10.6)
      WRITE(3,3104)P103
3104 FORMAT('LOAD003,2,',F10.6)
      WRITE(3,3105)P104
3105 FORMAT('LOAD004,2,',F10.6)
      WRITE(3,3200)P105
3200 FORMAT('LOAD005,2,',F10.6)
      WRITE(3,3201)P107
3201 FORMAT('LOAD006,2,',F10.6)
3204 FORMAT('119,2,',F10.6)
      WRITE(3,3205)
3205 FORMAT('**'/
     *,'**LEVEL=1, X=0'/
     *,'**')
      WRITE(3,3206)P2101
3206 FORMAT('2101,2,',F10.6)
      WRITE(3,3207)P2102
3207 FORMAT('2102,2,',F10.6)
      WRITE(3,3208)P2103
```

```
3208 FORMAT('2103,2,',F10.6)
     WRITE(3,3209)P2104
3209 FORMAT('2104,2,',F10.6)
     WRITE(3,3210)P2105
3210 FORMAT('2105,2,',F10.6)
     WRITE(3,3220)
3220 FORMAT('**'/
    *,'**LEVEL=2, X=0'/
    *,'**')
     WRITE(3,3130)P4101
3130 FORMAT('LOAD021,2,',F10.6)
     WRITE(3,3131)P4102
3131 FORMAT('LOAD022,2,',F10.6)
     WRITE(3,3132)P4106
3132 FORMAT('LOAD023,2,',F10.6)
      WRITE(3,3133)P4107
3133 FORMAT('LOAD024,2,',F10.6)
      WRITE(3,3134)P4111
3134 FORMAT('LOAD025,2,',F10.6)
      WRITE(3,3135)P4112
3135 FORMAT('LOAD026,2,',F10.6)
      WRITE(3,3136)P4116
3136 FORMAT('LOAD027,2,',F10.6)
      WRITE(3,3137)P4117
3137 FORMAT('LOAD028,2,',F10.6)
      WRITE(3,3138)P4121
3138 FORMAT('LOAD029,2,',F10.6)
      WRITE(3,3139)P4122
3139 FORMAT('LOAD0210,2,',F10.6)
     WRITE(3,3250)P4103
3250 FORMAT('4103,2,',F10.6)
     WRITE(3,3251)P4108
3251 FORMAT('4108,2,',F10.6)
     WRITE(3,3252)P4113
3252 FORMAT('4113,2,',F10.6)
     WRITE(3,3253)P4118
3253 FORMAT('4118,2,',F10.6)
     WRITE(3,3254)P4123
3254 FORMAT('4123,2,',F10.6)
     WRITE(3,3255)
3255 FORMAT('**'/
    *,'**LEVEL=0, X=L'/
    *,'**')
     WRITE(3,4101)P1301
4101 FORMAT('LOADL01,2,',F10.6)
     WRITE(3,4102)P1302
4102 FORMAT('LOADL02,2,',F10.6)
     WRITE(3,4103)P1303
4103 FORMAT('LOADL03,2,',F10.6)
     WRITE(3,4104)P1304
4104 FORMAT('LOADL04,2,',F10.6)
     WRITE(3,4200)P1305
4200 FORMAT('LOADL05,2,',F10.6)
     WRITE(3,4201)P1307
4201 FORMAT('LOADL06,2,',F10.6)
     WRITE(3,4205)
4205 FORMAT('**'/
    *,'**LEVEL=1 X=L'/
    *,'**')
     WRITE(3,4206)P3301
4206 FORMAT('3301,2,',F10.6)
     WRITE(3,4207)P3302
4207 FORMAT('3302,2,',F10.6)
     WRITE(3,4208)P3303
4208 FORMAT('3303,2,',F10.6)
     WRITE(3,4209)P3304
4209 FORMAT('3304,2,',F10.6)
```

```
      WRITE(3,4210)P3305
 4210 FORMAT('3305,2,',F10.6)
      WRITE(3,4220)
 4220 FORMAT('**'/
     *,'**LEVEL=2, X=L'/
     *,'**')
      WRITE(3,4130)P5301
 4130 FORMAT('LOADL21,2,',F10.6)
      WRITE(3,4131)P5302
 4131 FORMAT('LOADL22,2,',F10.6)
      WRITE(3,4132)P5306
 4132 FORMAT('LOADL23,2,',F10.6)
      WRITE(3,4133)P5307
 4133 FORMAT('LOADL24,2,',F10.6)
      WRITE(3,4134)P5311
 4134 FORMAT('LOADL25,2,',F10.6)
      WRITE(3,4135)P5312
 4135 FORMAT('LOADL26,2,',F10.6)
      WRITE(3,4136)P5316
 4136 FORMAT('LOADL27,2,',F10.6)
      WRITE(3,4137)P5317
 4137 FORMAT('LOADL28,2,',F10.6)
      WRITE(3,4138)P5321
 4138 FORMAT('LOADL29,2,',F10.6)
      WRITE(3,4139)P5322
 4139 FORMAT('LOADL210,2,',F10.6)
      WRITE(3,4250)P5303
 4250 FORMAT('5303,2,',F10.6)
      WRITE(3,4251)P5308
 4251 FORMAT('5308,2,',F10.6)
      WRITE(3,4252)P5313
 4252 FORMAT('5313,2,',F10.6)
      WRITE(3,4253)P5318
 4253 FORMAT('5318,2,',F10.6)
      WRITE(3,4254)P5323
 4254 FORMAT('5323,2,',F10.6)
      WRITE(3,4255)
 4255 FORMAT('**'/
     *,'*NODE PRINT'/
     *,'U'/
     *,'*FILE FORMAT, ASCII'/
     *,'*END STEP')
      RETURN
      END
C
C.... This subroutine returns the pdf of normal distribution....
C Input
C   x     : random variable
C   mean  : mean value
C   sigma : standard deviation
C Output
C   pdf   : Probability density function
C
      SUBROUTINE norpdf(x,mean,sigma,pdf)
      REAL*8 x,mean,sigma,pdf
      REAL*8 pi,z
      DATA pi/3.1415926535898/
      z=(x-mean)/sigma
      pdf=DEXP(-(z*z/2.))/(sigma*DSQRT(2.*pi))
      RETURN
      END
C
C....  This subroutine returns the cdf of normal distribution.....
C  The range of xx should be restricted between -13.0 - +13.0
C  Input
C     x  - random variable
```

```
C   mean - mean value
C  sigma - standard deviation
C  Output
C   cdf  - cummulative distribution function
C    iw  - warning message,
C    iw = 0 - ok ;
C    iw = 1 - warning, x is out of range
C
      SUBROUTINE norcdf(X,MEAN,SIGMA,CDF,IW)
      REAL*8 X,MEAN,SIGMA,CDF
      REAL*8 PA(8),QA(9),XX,Y,Z,YABS,PNUM,QDEN

      XX=(X-MEAN)/SIGMA
      IF(ABS(XX).GT.13.) THEN
        IW=1
      ELSE
        IW=0
      ENDIF
      PA(1)=883.47894260849
      PA(2)=1549.6793124037
      PA(3)=1347.1941340976
      PA(4)=723.04000277753
      PA(5)=255.50049469496
      PA(6)=59.240010112914
      PA(7)=8.376531081419699
      PA(8)=.56418955944261
      QA(1)=883.4789426085
      QA(2)=2546.5785458098
      QA(3)=3337.2213699893
      QA(4)=2606.7120152651
      QA(5)=1333.569975678
      QA(6)=460.2851236916
      QA(7)=105.50025439769
      QA(8)=14.847012237523
      QA(9)=1.
      Y=XX/DSQRT(2D+00)
      IF (DABS(Y).LT.1D-75) THEN
        CDF=.5
        RETURN
      ENDIF
      Z=DEXP(-Y**2)
      YABS=DABS(Y)
      PNUM=PA(8)
      DO 10 I=1, 7
          INDX=8-I
          PNUM=PA(INDX)+YABS*PNUM
10    CONTINUE
      QDEN=QA(9)
      DO 20 I=1, 8
          INDX=9-I
          QDEN=QA(INDX)+YABS*QDEN
20    CONTINUE
      CDF=(PNUM/QDEN)*.5*Z
      IF (XX.LT.0) RETURN
      CDF=1.-CDF
      RETURN
      END
C
C..... This subroutine returns the inverse value (x, r.v.) of cdf....
C       Input :
C            cdf : cumulative distribution function
C           mean : mean value
C       Output :
C             x   : random value
C
        SUBROUTINE norinv(x, mean, sigma, cdf)
        REAL*8 x,mean,sigma,cdf
```

```
      REAL*8 p,xp,p1,t1,t2,t3,t4,d1,d2

      p = 1. - cdf
      IF (p .EQ. .5) THEN
              XP = 0.
              X = mean
              RETURN
      ENDIF
      IF (p .GT. .5) THEN
              p1 = cdf
      ELSE
              p1 = p
      ENDIF
      t1 = p1**2
      t2 = 1/t1
      t3 = DLOG(t2)
      t4 = DSQRT(t3)
      d1 = 2.515517 + .802853 * t4 + .010328 * t4**2
      d2 = 1.432788 * t4 + .189269 * t4**2 + .001308 * t4**3
      xp = t4 - d1 / (1. + d2)
      IF (p .LT. .5) THEN
         x = xp * sigma + mean
      ELSE
         x = -xp * sigma + mean
      ENDIF
      RETURN
      END
```

## A.2. READIN

```
PROGRAM POST
C
C     GETING THE EIGENVALUE NUMBER AND THE EIGENVALUE
C
      IMPLICIT REAL*8 (A-H,O-Z)
      CHARACTER*80 FNAME
      DOUBLE PRECISION ARRAY
      DIMENSION ARRAY(513), JRRAY(2,513),LRUNIT(1,1)
      EQUIVALENCE (ARRAY(1),JRRAY(1,1))
      NRU=1
      LRUNIT(1,NRU)=8
      LRUNIT(2,NRU)=1
      LOUTF=0
C.......................................................................
C     OPEN OUTPUT FILE
C.......................................................................
C     OPEN(UNIT=15,FILE='eignv.out',STATUS='UNKNOWN')
C.......................................................................
C     PROMPT USER FOR EIGENVALUE NUMBER
C
      JEIGNO=1
      IF(JEIGNO.EQ.0) GOTO 200
C.......................................................................
C     ACCESS ABAQUS LIBRARY TO SET INPUT FILE.
C.......................................................................
      CALL INITPF(FNAME,NRU,LRUNIT,LOUTF)
      JUNIT=LRUNIT(1,NRU)
      CALL DBRNU(JUNIT)
      DO 100 K1=1,99999
      CALL DBFILE (0,ARRAY,JRCD)
      IF(JRCD.NE.0) go to 200
      IF (JRRAY(1,2) . EQ. 1980) THEN
           IEIGNO = ARRAY(3)
         EIGENV=ARRAY(4)
           WRITE(*,*) 'EIGENV =',EIGENV
```

```
      END IF
 100  CONTINUE
 200  CONTINUE
      STOP
      END
```

## A.3. STRENGTH

```
PROGRAM STRENGTH
      REAL*8 meanY,sigmaY
      REAL*8 x,mean,sigma,cdf
      REAL*8 FACTOR, EIGENV(1),HOLD,STRENGTH,Pf
      INTEGER NDIST
      OPEN(UNIT=1,FILE='FACTOR',STATUS='UNKNOWN')
      OPEN(UNIT=2,FILE='EIGENV',STATUS='UNKNOWN')
      READ(1,*) FACTOR
      PRINT*,'FACTOR =',FACTOR
C
C......SELECTION OF SMALLEST EIGEN VALUE.....
C
      OPEN(UNIT=3,FILE='load.inp',STATUS='UNKNOWN')
      REWIND(UNIT=3)
C
C.....N is number of calculated eigenvalues.....
C
      READ(3,*) NDIST,MEAN,SIGMA,N
      READ(2,*) (EIGENV(I), I=1,N)
      PRINT*,'EIGENV =',(EIGENV(I), I=1,N)
      REWIND(UNIT=2)
      READ(2,*) HOLD
      DO 10 I=1,N
                PRINT*,'EIGENV =',EIGENV(I)
                IF(HOLD.LT.EIGENV(I)) GO TO 10
             HOLD = EIGENV(I)
                PRINT*,'HOLD =',HOLD
  10  CONTINUE
                PRINT*,'HOLD =',HOLD
      STRENGTH=FACTOR*HOLD
      X = STRENGTH
      PRINT*,'STRENGTH =',STRENGTH
C
C.............DISTRIBUTIONS INPUT FILE............
C
C............. DISTRIBUTIONS................
C
C...2- LOGNORMAL DISTRIBUTION.
C
      PRINT*,'SIGMA =',SIGMA
      CALL logpar(mean,sigma,meanY,sigmaY)
      IF (NDIST.EQ.2) THEN
          CALL logcdf(x,meanY,sigmaY,cdf)
      ENDIF
      Pf=1-CDF
      PRINT*,'Pf      =',Pf
      END
C
C.... This subroutine returns the value of lognormal cdf.....
C   Input
C      x   : random value
C    mean : mean value (parameter)
C    sigma:  standard deviation (parameter)
C   Output
C      cdf : cumulative distribution function
C
      SUBROUTINE logcdf(x,meanY,sigmaY,cdf)
      REAL*8 x,meanY,sigmaY,cdf
```

```
      REAL*8 xx
      REAL*8 pa(9),qa(9),y,z,yabs,pnum,qden,temp

      xx =(DLOG(X)-meanY)/sigmaY

      pa(1) = 883.47894260849
      pa(2) = 1549.6793124037
      pa(3) = 1347.1941340976
      pa(4) = 723.04000277753
      pa(5) = 255.50049469496
      pa(6) = 59.240010112914
      pa(7) = 8.376531081419699
      pa(8) = .56418955944261
C
      qa(1) = 883.4789426085
      qa(2) = 2546.5785458098
      qa(3) = 3337.2213699893
      qa(4) = 2606.7120152651
      qa(5) = 1333.569975678
      qa(6) = 460.2851236916
      qa(7) = 105.50025439769
      qa(8) = 14.847012237523
      qa(9) = 1.
C
      temp=2.0
      y = xx / DSQRT(temp)
      IF (DABS(y) .LT. 1D-75) THEN
        cdf = .5
        RETURN
      ENDIF
C
      z = DEXP(-y**2)
      yabs = DABS(y)
      pnum = pa(8)
C
      DO 100 I = 1, 7
          indx = 8 - I
          pnum = pa(indx) + yabs * pnum
100   CONTINUE
C
      qden = qa(9)
      DO 200 I = 1, 8
          indx = 9 - I
          qden = qa(indx) + yabs * qden
200   CONTINUE
C
      cdf = (pnum / qden) * .5 * z
      IF (xx .LT. 0.) RETURN
      cdf = 1 - cdf
      RETURN
      END
C
C.... This subroutine returns the lognormal parameters meanY & sigmaY..
C     Input
C      meanX :  mean value of X
C     sigmaX :  standard deviation of X
C     Output
C      meanY :  mean value of Y
C      sigmaY:  standard deviation of Y
C
      SUBROUTINE logpar(meanX,sigmaX,meanY,sigmaY)
      REAL*8 meanX,sigmaX,meanY,sigmaY
      REAL*8 cvX,varY

      cvX=sigmaX/meanX
      varY=DLOG(1.+cvX*cvX)
      sigmaY=DSQRT(varY)
```

```
      meanY=DLOG(meanX)-varY/2.
      RETURN
      END
```

# A.4. STATISTICS

```
PROGRAM STATISTICS
      REAL*8 Pf,SUMPf,AVEPf,SUMPf2,SDPf,COVPf,A,B,C,D
      INTEGER N
      OPEN(UNIT=1,FILE='cycle',STATUS='unknown')
      READ(1,*) N
      PRINT*,'-----------------------------------------------'
      PRINT*,'CYCLE No.=',N
      PRINT*,'-----------------------------------------------'
      OPEN(UNIT=2,FILE='PFAIL',STATUS='unknown')
      READ(2,*) Pf
      PRINT*,'Pf      =',Pf
      OPEN(UNIT=3,FILE='SUMPf',STATUS='unknown')
      READ(3,*) SUMPf
      SUMPf=SUMPf+Pf
      PRINT*,'SUMPf   =',SUMPf
      AVEPf=SUMPf/N
      PRINT*,'AVEPf   =',AVEPf
      REWIND(UNIT=3)
      WRITE(3,*) SUMPf
C
      OPEN(UNIT=4,FILE='SUMPf2',STATUS='unknown')
      READ(4,*) SUMPf2
      SUMPf2=SUMPf2+Pf**2
      PRINT*,'SUMPf2 =',SUMPf2
      REWIND(UNIT=4)
      WRITE(4,*) SUMPf2
      C= 1./(N*REAL(N-1))
      A = DSQRT(C)
      D=SUMPf2-N*AVEPf**2
      B = DSQRT(D)
      SDPf=A*B
      PRINT*,'SDPf    =',SDPf
      COVPf=SDPf/AVEPf
      PRINT*,'COVPf   =',COVPf
      END
```